Similarity Solution for Laminar Boundary Layer over a Semi-Infinite Vertical Flat Plate: Uniform Heat Flux

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Abstract

This study presents a comprehensive similarity solution for laminar boundary layer flow over a semi-infinite vertical flat plate subjected to uniform heat flux (UHF) boundary conditions. The analysis begins with the formulation of governing equations for momentum and energy conservation, incorporating the Boussinesq approximation to account for buoyancy effects. Through similarity transformations, the partial differential equations are reduced to coupled ordinary differential equations, which are solved numerically for various Prandtl numbers (0.1 to 100). Key dimensionless parameters including the modified Grashof number (Gr_x^*) and Nusselt number (Nu_x) are derived, revealing the characteristic $x^{1/5}$ dependence of surface temperature and $x^{4/5}$ scaling of local heat transfer coefficient. A practical example demonstrates the iterative solution methodology for a 20 cm vertical plate in air, showing excellent convergence after three iterations with a maximum surface temperature of 393.5°C at the trailing edge. The results are validated against established correlations from Sparrow and Gregg, with deviations within 0.55%. The derived relationships for surface temperature distribution, local Nusselt number, and average heat transfer coefficient provide valuable information regarding the engineering design of vertical surfaces under constant heat flux conditions.

Analysis

One may consider the phenomenon of free convection in quiescent fluid over vertical plate at temperature T_∞. The plate is subjected to constant uniform heat flux (UHF) $q_w^{"}$. The uniform heat flux is much realistic as compared to uniform wall temperature boundary condition. The constant heat flux $q_w^{"}$ can be generated through electric heating of surface. The fluid properties remain the same except for fluidic density close to the plate. The density is accounted for as an important variable in the formulation of buoyancy term in the momentum equation. The uniform heat flux was solved by Sparrow and Gregg (1956) using similarity transformation for various Prandtl numbers. The similarity solution begins by choosing the stream functions Ψ:

$$u = \frac{\partial \Psi}{\partial y} \tag{1}$$

$$v = \frac{-\partial \Psi}{\partial x} \tag{2}$$

Thus the continuity equation is satisfied. The energy equation and corresponding boundary conditions are given as follows:

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \alpha \frac{\partial^2 \theta}{\partial y^2} \tag{3}$$

The boundary conditions are as follows:

$$x = 0 \qquad \theta = 0 \tag{4}$$

$$y = 0 \qquad -k\frac{\partial \theta}{\partial y} = q_w^{"} \tag{5}$$

$$y = \infty \qquad \theta = 0 \tag{6}$$

Where $\theta = T - T_\infty$ is the temperature. The following variables have been used in order to transform the partial differential equations into ordinary differential equations:

$$\eta = \frac{C_1 y}{x^{\frac{1}{5}}} \tag{7}$$

where

$$C_1 = (\frac{g\beta q_w^{"}}{5k\nu^2})^{\frac{1}{5}}$$

where η is the similarity variable. The stream function Ψ is given as:

$$\Psi = C_2 x^{\frac{4}{5}} F(\eta) \tag{8}$$

where $F(\eta)$ is the dimensionless velocity function and C_2 is given as:

$$C_2 = (\frac{5^4 \beta g q_w^{"} \nu^3}{k})^{\frac{1}{5}}$$

The dimensionless temperature parameter θ is given as:

$$\theta = \frac{C_1 (T - T_\infty)}{x^{\frac{1}{5}} (\frac{q_w^{"}}{k})} \tag{9}$$

The horizontal and vertical components of velocity are given as follows:

$$u = 5\nu C_1^2 x^{\frac{3}{5}} \frac{dF}{d\eta} \tag{10}$$

$$v = \nu C_1 x^{\frac{-1}{5}} [\eta \frac{dF}{d\eta} - 4F] \tag{11}$$

Using the above equations and similarity variables, the momentum and energy equations are successfully transformed into ordinary differential equations through algebraic manipulation. All the factors incorporating the variable "x" tend to cancel out, leaving behind only the variables dependent upon η. The final equations are as follows: Momentum equation:

$$\frac{d^3 F}{d\eta^3} + 4F \frac{d^2 F}{d\eta^2} - 3(\frac{dF}{d\eta})^2 + \theta = 0 \tag{12}$$

$$F(0) = 0 \tag{13}$$

$$F'(0) = 0 \tag{14}$$

$$F'(\infty) = 0 \tag{15}$$

Energy Equation:

$$\frac{d^2 \theta}{d\eta^2} + Pr[4F \frac{d\theta}{d\eta} - \frac{dF}{d\eta}\theta] = 0 \tag{16}$$

$$\theta'(0) = -1 \tag{17}$$

$$\theta(\infty) = 0 \tag{18}$$

The primes in the conditions indicate the differentiation with respect to η. The dependent variables $F(\eta)$ and $\theta(\eta)$ are coupled together. Simultaneous solution of such equations is thus required. Solutions for Prandtl number 0.1, 1.0, 10 and 100 are obtained by Sparrow and Gregg (1956). The uniform surface heat flux, the plate surface temperature tend to vary along the plate. The surface temperature variation $T_w(x)$ and the local Nusselt number Nu_x have to be determined. The following table 1 provides the numerical solution data: The surface temperature $T_w(x)$ is obtained through the following equation:

$$\begin{aligned} \mathrm{T_w(x) - T_\infty} &= \frac{\mathrm{q''_w}}{\mathrm{k}} \left(\frac{\mathrm{x}^{1/5}}{\mathrm{C_1}} \right) \theta(0) = \frac{\mathrm{q''_w\, x^{1/5}}}{\mathrm{k}} \frac{\theta(0)}{\left[\left(\frac{\mathcal{B}\, \bar{\mathrm{P}}\, \mathrm{q''_w}}{5\mathrm{k}\, \nu^2} \right) \right]^{1/5}} \\ &= \frac{\mathrm{q''_w\, x^{1/5}}}{\mathrm{k}} \frac{\theta(0)}{\left[\left(\frac{\mathcal{B}\, \bar{\mathrm{P}}\, \mathrm{q''_w x^4}}{\mathrm{k}\, \nu^2} \frac{1}{5\mathrm{x}^4} \right) \right]^{1/5}} = \frac{\mathrm{q''_w\, x^{1/5}}}{\mathrm{k}} \frac{\theta(0)\, (5\mathrm{x}^4)^{1/5}}{\left[\left(\frac{\mathcal{B}\, \bar{\mathrm{P}}\, \mathrm{q''_w x^4}}{\mathrm{k}\, \nu^2} \right) \right]^{1/5}} \\ &= 5^{1/5} \frac{\mathrm{q''_w\, x}}{\mathrm{k}} \frac{1}{(\mathrm{Gr^\star_x})^{1/5}} \theta(0) \end{aligned} \tag{19}$$

Table 1: Numerical results for different Prandtl numbers

2*Pr	Sparrow and Gregg [8]		Maple 2020	
	$\theta(0)$	$F''(0)$	$\theta'(0)$	$F''(0)$
0.1	−2.7507	1.6434		
0.72			1.48785	0.81168
1	−1.3574	0.72196	1.3586	0.7201
2			1.1315	0.55910
5			0.90346	0.39739
10	−0.76746	0.30639	0.76955	0.30589
100	−0.46566	0.12620	0.46798	0.12562

Where Gr_x^* is the modified Grashof number suggested by Sparrow and Gregg and is given as:

$$Gr_x^* = \frac{g\beta q_w^{"} x^4}{k\nu^2} \tag{20}$$

In terms of modified Grashof number Gr_x^*, the becomes:

$$\frac{(T_w(x) - T_\infty)}{(\frac{q_w^{"} x}{k})} = 5^{\frac{1}{5}}\theta(0) \tag{21}$$

The modified Grashof number is required since the temperature difference $[T_w(x) - T_\infty]$ is unknown initially and it tends to vary with x along the vertical plate. The local heat transfer coefficient $h(x)$ can be evaluated as:

$$h = \frac{q_w^{"}}{T_w - T_\infty} = \frac{q_w^{"}}{5^{\frac{1}{5}} \frac{q_w^{"} x}{k} \frac{\theta(0)}{(Gr_x^*)^{\frac{1}{5}}}} = \frac{k}{x}\frac{(\frac{Gr_x^*}{5})^{\frac{1}{5}}}{\theta(0)} \tag{22}$$

The local Nusselt number Nu_x is given as:

$$Nu_x = \frac{hx}{k} = \frac{(\frac{Gr_x^*}{5})^{\frac{1}{5}}}{\theta(0)} = \frac{1}{5^{\frac{1}{5}}}\frac{(Gr_x^*)^{\frac{1}{5}}}{\theta(0)} \tag{23}$$

The negative sign from Sparrow and Grieg is the definition of dimensionless temperature. the defined dimensionless temperature function is given as:

$$\theta = \frac{C_1(T_\infty - T)}{x^{\frac{1}{5}}(\frac{q_w^{"}}{k})} \tag{24}$$

Study conducted by Fuji and Fuji (1976) developed an expression in terms of dimensionless parameter $\theta(0)$ which can be estimated using the following correlation:

$$\theta(0) = [\frac{4 + 9\sqrt{Pr} + 10Pr}{5Pr^2}]^{\frac{1}{5}}(Gr_x^* Pr)^{\frac{1}{5}} \tag{25}$$

Where $Gr_x^* = \frac{g\beta q_w^{"} x^4}{k\nu^2}$ is the modified Grashof number. Burmesiter Burmeister (1993) reported the flow to be laminar provided following condition is met:

$$Gr_x^* \leq (\frac{0.916Pr}{Pr + 0.8})^{\frac{1}{4}} \times 10^{11}$$

The modified Grashof number $Gr_x^* = \frac{g\beta q_w^{"} x^4}{k\nu^2}$ is related to the conventional Grashof number $Gr_x = \frac{g\beta(T_w - T_\infty)x^3}{\nu^2}$ by

$$Gr_x^* = Gr_x Nu_x$$

since $q_w^{"} = g\beta(T_w - T_\infty)$. Following the research conducted by La Fevre , White (1991) reported curve fit formula for numerical results with an accuracy of ±0.55% The Nusselt number is given by:

$$\mathrm{Nu}_x = \frac{\mathrm{Pr}^{2/5}}{\left(3.91 + 9.32\sqrt{\mathrm{Pr}} + 9.95\ \mathrm{Pr}\right)^{1/5}} \left(\mathrm{Gr}_x^*\right)^{1/5}. \tag{26}$$

Once we have Nu_x, we can find the local temperature difference:

$$T_w(x) - T_\infty = \frac{q''_w x}{k} \frac{1}{\mathrm{Nu}_x}. \tag{27}$$

Using Eq. (26) in Eq. (27):

$$T_w(x) - T_\infty = \frac{(q''_w x)}{k} \left[\frac{4 + 9\sqrt{\mathrm{Pr}} + 10\ \mathrm{Pr}}{\mathrm{Pr}} \right]^{1/5} \frac{1}{(\mathrm{Pr}\ \ \mathrm{Gr}_x^*)^{1/5}} \tag{28}$$

$$= \left[\frac{4 + 9\sqrt{\mathrm{Pr}} + 10\ \mathrm{Pr}}{\mathrm{Pr}^2} \left(\frac{\nu^2}{g\beta} \right) \left(\frac{q''_w}{k} \right)^4 x \right]^{1/5}. \tag{29}$$

For a vertical plate with constant heat flux:

$$q''_w = \text{const} \tag{30}$$

$$10^4 < \mathrm{Gr}_x^*\ \mathrm{Pr} < 10^9 \tag{31}$$

$$0 \leq \mathrm{Pr} \leq \infty. \tag{32}$$

The average temperature difference is:

$$\overline{T}_w - \overline{T}_\infty = \frac{1}{L} \int_0^L (T_w - T_\infty) \mathrm{d}x = \frac{5}{6} (T_w - T_\infty)_{x=L}. \tag{33}$$

Evaluating at $x = L$:

$$(T_w - T_\infty)_{x=L} = 5^{1/5} \frac{q''_w}{k} \frac{L}{(\mathrm{Gr}_L^*)^{1/5}} \theta(0). \tag{34}$$

The average temperature becomes:

$$\left(\overline{T}_w - \overline{T}_\infty\right) = \left(\frac{5}{6} \right) 5^{1/5} \frac{q''_w}{k} \frac{L}{(\mathrm{Gr}_L^*)^{1/5}} \theta(0) = \left(\frac{5^{6/5}}{6} \right) \frac{q''_w}{k} \frac{L}{(\mathrm{Gr}_L^*)^{1/5}}. \tag{35}$$

The average heat transfer coefficient is:

$$\bar{h} = \frac{q''_w}{\overline{T}_w - \overline{T}_\infty} = \frac{q''_w}{\left(\frac{5^{6/5}}{6} \right) \frac{q''_w}{k} \frac{L}{(\mathrm{Gr}_L^*)^{1/5}}}. \tag{36}$$

The average Nusselt number is:

$$\overline{\mathrm{Nu}}_L = \frac{q''_w L}{k(\overline{T}_w - \overline{T}_\infty)} = \left[\frac{6}{5^{6/5}} \frac{1}{\theta(0)} \right] (\mathrm{Gr}_L^*)^{1/5}. \tag{37}$$

The modified Grashof number relations:

$$\mathrm{Gr}_L = \frac{g\beta(\overline{T}_w - \overline{T}_\infty) L^3}{\nu^2}. \tag{38}$$

$$\mathrm{Gr}_L^* = \frac{g\,\beta\,q''_w\,L^4}{k\,\nu^2} = \frac{g\beta(\overline{T}_w - \overline{T}_\infty) L^3}{\nu^2} \frac{q''_w L}{k} = \mathrm{Gr}_L \mathrm{Nu}_L. \tag{39}$$

Substituting into the Nusselt number relation:

$$\overline{\mathrm{Nu}}_L = \left[\frac{6}{5^{6/5}} \frac{1}{\theta(0)} \right] (\mathrm{Gr}_L \mathrm{Nu}_L)^{1/5} \tag{40}$$

or after rearrangement:

$$\overline{\mathrm{Nu}}_L = \left[\frac{6}{5^{6/5}} \frac{1}{\theta(0)} \right]^{5/4} (\mathrm{Gr}_L)^{1/4}. \tag{41}$$

Practical Example

A 20-cm-high and 10-cm-wide flat plate is placed in a vertical position in air at 1-atm pressure and 17 °C. The back side of the plate is thermally insulated, and the thickness of the plate is negligible. The plate is exposed to heat flux of $q_w^{"} = 2500 W/m^2$ on the front side. The energy is dissipated through free convection.
Evaluate:
(a) the maximum temperature on the plate
(b) variation of the surface temperature of the plate.
(c) average heat transfer coefficient. Onur (2023).

Solution Implementation

The surface temperature is not known and the film temperature T_f cannot be determined to evaluate the air properties. We convert the air temperature to Kelvin, and as a first approximation, we evaluate the air properties at this temperature:

$$T_\infty = 17\,^\circ\mathrm{C} = 290\,\mathrm{K}. \tag{42}$$

First iteration:

The properties of air at this temperature are taken from the air table:

$$\nu = 15.0 \times 10^{-6}\,\mathrm{m}^2/\mathrm{s}, \quad k = 25.5 \times 10^{-3}\,\mathrm{W/m \cdot K}, \quad \mathrm{Pr} = 0.709. \tag{43}$$

We evaluate the coefficient of thermal expansion at the film temperature T_∞:

$$\beta = \frac{1}{T_\infty} = \frac{1}{290} = 0.00344\,\mathrm{K}^{-1} \tag{44}$$

Next, we evaluate the modified Grashof number Gr_x^* as:

$$\mathrm{Gr}_x^* = \frac{g\beta q_w'' x^4}{k\nu^2} = \frac{9.81 \times \left(\frac{1}{280}\right)(2500)x^4}{(0.0255)(15.0 \times 10^{-6})^2} = 1.473 \times 10^{13} x^4. \tag{45}$$

Next, we compute the local Nusselt number Nu_x using Eq. (12.69b):

$$\mathrm{Nu}_x = \frac{\mathrm{Pr}^{2/5}}{(3.91 + 9.32\sqrt{\mathrm{Pr}} + 9.95\,\mathrm{Pr})^{1/5}} (\mathrm{Gr}_x^*)^{1/5} = 208.45\, x^{4/5}. \tag{46}$$

Then, we write the surface temperature $T_w(x)$ as:

$$T_w(x) = T_\infty + \frac{q_w'' x}{k} \frac{1}{\mathrm{Nu}_x} \tag{47}$$

$$T_w(x) = 17 + \frac{2500\, x}{0.00255} \frac{1}{208.45\, x^{4/5}} \tag{48}$$

$$T_w(x) \approx 17 + 470.32\, x^{1/5} \tag{49}$$

The temperature values for the first trial are given in Table 2.

Table 2: Wall temperature distribution (First iteration)

x (cm)	0	2	4	6	8	10	12	14	16	18	20
T_w (°C)	17	232.0	264.0	284.9	300.7	313.7	324.7	334.4	342.9	350.7	357.8

The maximum temperature occurs at the trailing edge, $x = 0.20$ m:

$$T_w\big|_{\max} \approx 17 + 470.32\,(0.2)^{1/5} \approx 357.8\,^\circ\mathrm{C} \tag{50}$$

The average wall temperature is:

$$\overline{T}_w = \frac{1}{L}\int_0^L T_w(x)dx = \frac{1}{0.20}\int_0^{0.20} [17 + 470.32\, x^{1/5}]\, dx \approx 301.0\,^\circ\mathrm{C}. \tag{51}$$

Second iteration:

We now evaluate the film temperature T_f as:

$$T_f = \frac{\overline{T}_w + T_\infty}{2} = \frac{301.0 + 17}{2} = 159.2\,^\circ\text{C} \approx 432\,\text{K}. \tag{52}$$

We now evaluate air properties at $T_f \approx 432\,\text{K}$ as:

$$\nu = 30.23 \times 10^{-6}\,\text{m}^2/\text{s}, \quad k = 36.0 \times 10^{-3}\,\text{W/m} \cdot \text{K}, \quad \text{Pr} = 0.687 \tag{53}$$

$$\beta = \frac{1}{T_f} = \frac{1}{432} = 0.002314\,\text{K}^{-1}. \tag{54}$$

The Grashof number is:

$$\text{Gr}_x^* = \frac{g\beta q_w'' x^4}{k\nu^2} = \frac{9.81 \times \left(\frac{1}{432}\right)(2500)x^4}{(0.0360)(30.23 \times 10^{-6})^2} \approx 1.725 \times 10^{12} x^4. \tag{55}$$

The Local Nusselt number is:

$$\text{Nu}_x = 134.53\, x^{4/5}. \tag{56}$$

Wall temperature distribution is:

$$T_w(x) = 17 + \frac{2500\, x}{0.686 \cdot 134.53\, x^{4/5}} \tag{57}$$

The temperature values for the second iteration are given in Table 3.

Table 3: Wall temperature distribution (Second iteration)

x (cm)	0	2	4	6	8	10	12	14	16	18	20
T_w (°C)	17	253.0	288.1	311.0	328.4	342.7	354.8	365.3	374.8	383.3	391.1

The maximum temperature occurs at the trailing edge, $x = 0.20$ m:

$$T_w\big|_{\text{max}} = 17 + 516.2\,(0.2)^{1/5} = 391.1\,^\circ\text{C}. \tag{58}$$

The average surface temperature is:

$$\overline{T}_w = \frac{1}{L}\int_0^L T_w(x)dx = \frac{1}{0.20}\int_0^{0.20} [17 + 516.20\, x^{1/5}]\, dx \approx 328.7\,^\circ\text{C}. \tag{59}$$

Third iteration:

We now evaluate film temperature T_f as:

$$T_f = \frac{\overline{T}_w + T_\infty}{2} = \frac{328.7 + 17}{2} = 172.85\,^\circ\text{C} \approx 446\,\text{K}. \tag{60}$$

We evaluate the air properties at this new temperature:

$$\nu = 31.91 \times 10^{-6}\,\text{m}^2/\text{s}, \quad k = 37 \times 10^{-3}\,\text{W/m} \cdot \text{K}, \quad \text{Pr} = 0.686 \tag{61}$$

$$\beta = \frac{1}{T_f} = 0.002242\,\text{K}^{-1}. \tag{62}$$

The Grashof number is:

$$\text{Gr}_x^* = 1.459 \times 10^{12} x^4. \tag{63}$$

The Nusselt number is:

$$\text{Nu}_x = 130.07\, x^{4/5}. \tag{64}$$

Temperature distribution is:

$$T_w(x) = 17 + 519.47\, x^{1/5} \tag{65}$$

The temperature values for the third iteration are given in Table 4.

Table 4: Wall temperature distribution (Third iteration)

x (cm)	0	2	4	6	8	10	12	14	16	18	20
T_w (°C)	17	254.5	289.8	312.9	330.4	344.7	356.9	367.5	377.0	385.6	393.5

The maximum temperature occurs at the trailing edge, $x = 0.20$ m:

$$T_w\big|_{\max} = 17 + 519.47\,(0.2)^{1/5} = 393.5\,^\circ\text{C}. \tag{66}$$

The average surface temperature is:

$$\overline{T}_w = \frac{1}{L}\int_0^L T_w(x)dx = \frac{1}{0.20}\int_0^{0.20} [17 + 519.55\,x^{1/5}]\,dx = 330.7\,^\circ\text{C}. \tag{67}$$

```
clc
clear
% Constants and Plate Geometry
g = 9.81;                % gravitational acceleration (m/s^2)
L = 0.2;                 % plate height (m)
W = 0.1;                 % plate width (m)
qw = 2500;               % surface heat flux (W/m^2)

% Air Properties at ~290K
nu = 15e-6;              % kinematic viscosity (m^2/s)
k = 25.5e-3;             % thermal conductivity (W/m·K)
Pr = 0.709;              % Prandtl number

% Domain Ranges
x_vals = linspace(0.005, 0.2, 100);      % plate positions (m)
Tinf_vals = linspace(290, 310, 100);     % ambient temperatures (K)
% Meshgrid for 2D parameter sweep
[X, TINF] = meshgrid(x_vals, Tinf_vals);
% Local Grashof number GR(x)
GRX = (g * (1 ./ TINF) * qw .* X.^4) ./ (k * nu^2);
% Local Nusselt number Nu(x)
PAY = (Pr)^(2/5) * GRX.^(1/5);
PAYDA = (3.91 + 9.32 * sqrt(Pr) + 9.95 * Pr)^(1/5);
NUX = PAY ./ PAYDA;

% Wall Temperature TW(x)
TW = 17 + (qw .* X) ./ (k .* NUX);  % in °C

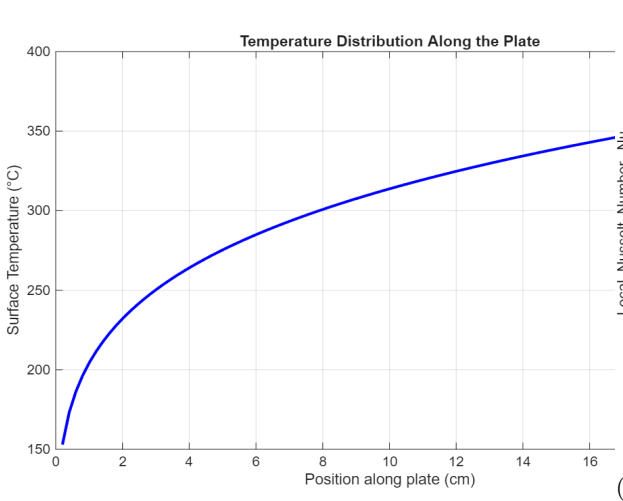
% Film Temperature
TF = (TW + (TINF - 273)) / 2;        % °C
% Average Wall Temperature (1D case at base Tinf)
x_1D = linspace(0.001, L, 1000);
TW_1D = 17 + 470.253 * x_1D.^(1/5);  % °C
TWA = trapz(x_1D, TW_1D) / L;
% Average Temperature
fprintf('Average Wall Temperature (TWA): %.2f °C\n', TWA);
% Plotting of Contours and Curves

% Wall Temperature along plate
figure;
plot(x_1D, TW_1D, 'b', 'LineWidth', 2);
xlabel('x (m)');
ylabel('Wall Temperature TW (°C)');
title('Wall Temperature Distribution Along Plate');
grid on;
```

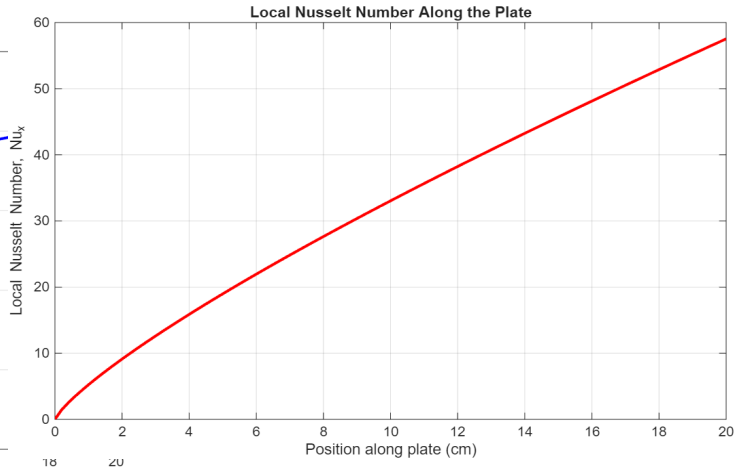
```
% Contour plot of Grashof number
figure;
contourf(X, TINF, GRX, 50, 'LineColor', 'none');
colorbar;
xlabel('x (m)');
ylabel('Ambient Temperature T_{} (K)');
title('Contour of Local Grashof Number Gr(x, T_{})');

% Contour plot of Wall Temperature
figure;
contourf(X, TINF, TW, 50, 'LineColor', 'none');
colorbar;
xlabel('x (m)');
ylabel('Ambient Temperature T_{} (K)');
title('Wall Temperature T_W(x, T_{})');

% Contour plot of Film Temperature
figure;
contourf(X, TINF, TF, 50, 'LineColor', 'none');
colorbar;
xlabel('x (m)');
ylabel('Ambient Temperature T_{} (K)');
title('Film Temperature T_F(x, T_{})');
% 3D Surface Plot of Wall Temperature
figure;
surf(X, TINF, TW);
xlabel('x (m)');
ylabel('T_{} (K)');
zlabel('T_W (°C)');
title('3D Surface: Wall Temperature');
shading interp;
colorbar;
```

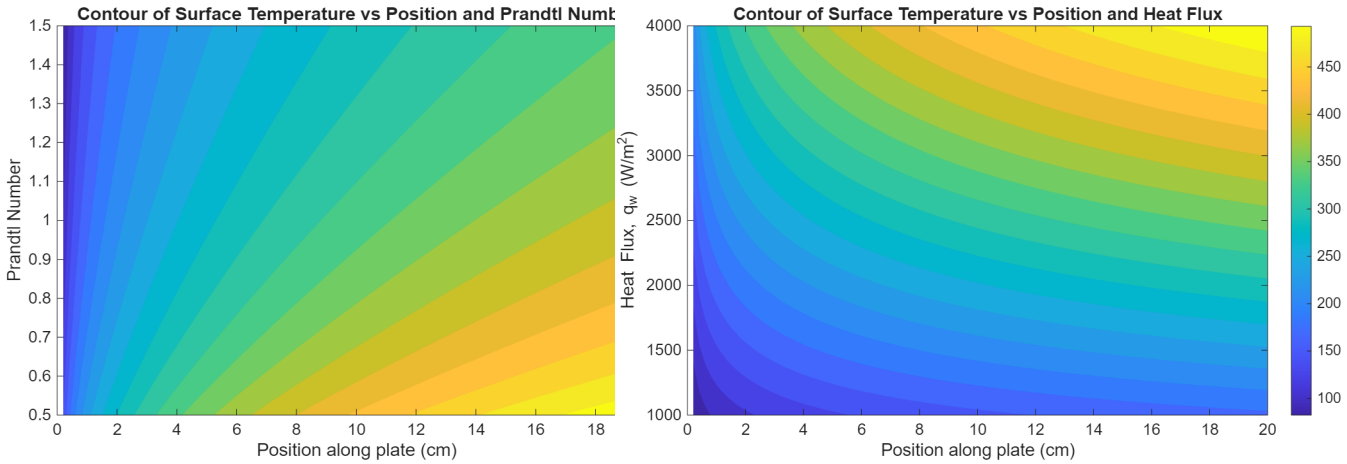


(a) Temperature Distribution along Vertical Plate



(b) Variation in Local Nusselt Number along Vertical Plate

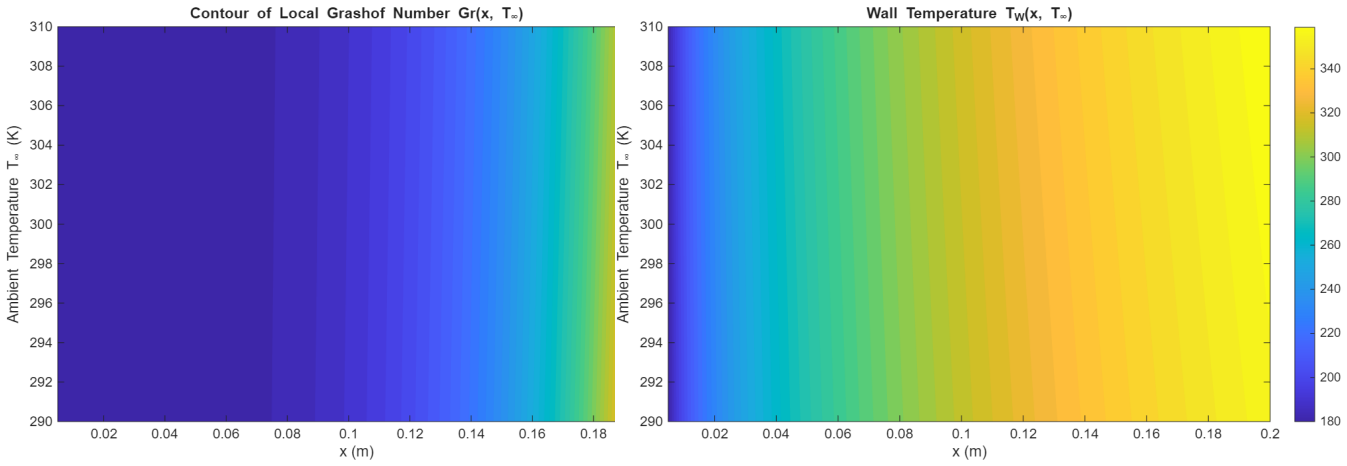
Figure 1: Plots for Temperature Distribution and Variation in Local Nusselt's Number along Vertical Plate



(a) Contour of Surface Temperature vs. Position and Prandtl Number

(b) Contour of Surface Temperature vs. Position and Heat Flux

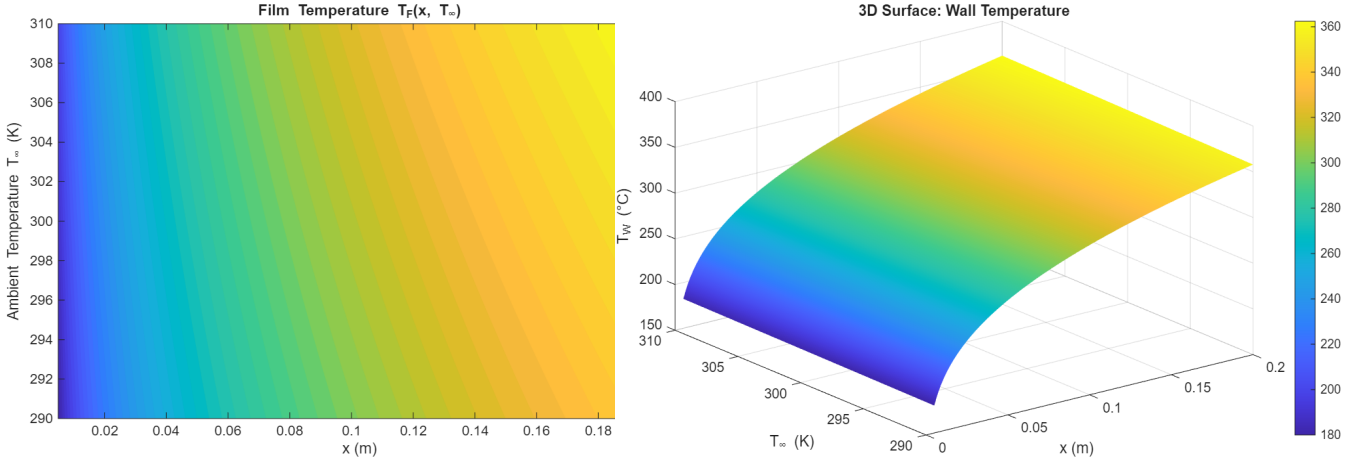
Figure 2: Contours of Surface Temperature vs. Position and Heat Parameters



(a) Contour of Local Grashof Number $Gr_x^*(x, T_\infty)$

(b) Contour of Wall Temperature $T_w(x, T_\infty)$

Figure 3: Contours of Local Grashof Number and Wall Temperature



(a) Contour of Film Temperature $T_F(x, T_\infty)$

(b) 3D Mesh of Wall Temperature Distribution

Figure 4: Film Temperature Contour and 3D Wall Temperature Mesh

Conclusion

This study provides a thorough theoretical and numerical analysis of laminar free convection over a vertical flat plate subjected to uniform heat flux conditions. The similarity solution approach successfully

transformed the governing boundary layer equations into a self-similar form, with key dimensionless parameters (0) and $F''(0)$ depending on the Prandtl number only. The analysis revealed characteristic scaling laws, with surface temperature varying as $x^{1/5}$ and local Nusselt number following $x^{4/5}$ dependence, consistent with established theoretical predictions. The practical case study demonstrated the effectiveness of the iterative solution methodology, showing rapid convergence to a maximum plate temperature of $393.5C$ at the trailing edge. The modified Grashof number Gr_x^* proved to be an essential parameter for characterizing the flow when the temperature difference is unknown, providing the critical link between conventional Grashof number and Nusselt number through the relation $Gr_x^* = Gr_x Nu_x$. The derived correlations, particularly for average Nusselt number showing the characteristic $Gr_L^{1/4}$ dependence, offer practical tools for engineering design applications. These findings have direct relevance for thermal management of electronic systems, solar panels, and other applications involving vertical heated surfaces. The study underscores the continued importance of similarity solutions in modern heat transfer analysis, especially when combined with iterative numerical techniques for accurate property evaluation. Future research directions could productively explore extensions to turbulent flow regimes and mixed convection scenarios.

References

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