Similarity Solution for Laminar Boundary Layer over a Semi-Infinite Vertical Flat Plate: Constant Wall Temperature

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Abstract

This study presents a comprehensive analysis of the laminar free convection boundary layer over a semi-infinite vertical flat plate maintained at a constant wall temperature. The governing partial differential equations, including continuity, momentum, and energy equations, are transformed into a set of ordinary differential equations using similarity variables. The similarity transformation utilizes the Grashof number and a dimensionless distance variable in order to reduce the system to coupled nonlinear ODEs, which are later solved numerically using MATLAB. The solution methodology is applied to a practical scenario involving a 40-cm-high and 20-cm-wide flat plate in air at 17°C with the plate surface temperature at 237°C. The velocity and temperature profiles at the mid-height of the plate devised and deliberated upon. In addition, the total heat loss from the plate is estimated using the average Nusselt number correlation for air (Prandtl number $Pr = 0.72$), yielding a heat transfer rate of 105.52 W. The results highlights the effectiveness of the similarity solution approach and align with established correlations for natural convection heat transfer.

Analysis

One may consider the phenomenon of free convection in quiescent fluid over vertical plate at temperature T_∞, where the plate temperature is denoted as $_w$ such that T_w¿T_∞ with constant overall temperature distribution. The fluid properties in general remain constant except the fluid density close to the hot plate. Using similarity method, the continuity, momentum and energy equations shall be solved where using the similarity transformations, the partial differential equations shall be transformed into ordinary differential equations Ostrach (1952),Arpaci and Larsen (1984). It is necessary to mention that the velocity and temperature profiles in different horizontal positions along the boundary layer remain geometrically same, varying by stretching factor along the x-direction Kays et al. (2005), Sissom and Pitts (1972). Pohlhausen, in collaboration with Schmidt and Beckmann, solved this set of boundary-layer differential equations for a vertical flat plate. In an examination of experimental data provided by Schimidt and Beckmann, it was observed by Pohlhausen that the velocity profiles along the plate were similar. He also observed that the temperature profiles were similar. Using the similarity of the velocity profiles and the similarity of temperature profiles at various locations on the vertical flat plate, Pohlhausen was able to obtain parameters to convert the partial differential equations into a pair of ordinary differential equations. The solution involves transformation variables by introducing a similarity variable of the form:

$$\eta = \frac{cy}{x^{\frac{1}{4}}} \tag{1}$$

where c is given by:

$$c = (\frac{g\beta\theta_w}{4\nu^2})^{\frac{1}{4}} \tag{2}$$

where $\theta_w = T_w - T_\infty$, g being the acceleration due to gravity, β being the thermal coefficient of expansion and ν is the kinematic viscosity. Using Grashof number Gr_x, the variable η can be expressed with substitution of c as:

$$\eta = \frac{y}{x^{\frac{1}{4}}}(\frac{g\beta\theta_w}{4\nu^2})^{\frac{1}{4}} = y(\frac{g\beta\theta_w x^3}{4\nu^2 x^4}) = \frac{y}{x}(\frac{Gr_x}{4})^{\frac{1}{4}} \tag{3}$$

where $Gr_x = \frac{g\beta(T_w - T_\infty)x^3}{\nu^2}$ is the localized Grashof number. The temperature can be expressed as nondimensionalized parameter as:

$$\theta = \frac{T - T_\infty}{T_w - T_\infty} \tag{4}$$

where the nondimensionalized temperature is dependent on the variable η:

$$\theta(x, y) = \eta(x, y) \tag{5}$$

Using stream functions, the velocity components can be expressed adequately as:

$$u = \frac{\partial \Psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \Psi}{\partial x} \tag{6}$$

The continuity equation is thus satisfied. For said analysis, he streamwise function can be defined as:

$$\Psi(x, y) = [4\nu(\frac{Gr_x}{4})^{\frac{1}{4}}]F(\eta) \tag{7}$$

Where $F(\eta)$ is the nondimensionalized velocity function. The substitution of in results in the following expressions:

$$u = 4\nu c^2\sqrt{x}\frac{dF}{d\eta} = 2\sqrt{x\beta g(T_w - T_\infty)}\frac{dF}{d\eta} = (\frac{2\nu}{x})\sqrt{Gr_x}\frac{dF}{d\eta} \tag{8}$$

$$v = 4\nu c x^{\frac{-1}{4}}[\eta\frac{dF}{d\eta} - 3F] = [\frac{g\beta\nu^2(T_w - T_\infty)}{4x}]^{\frac{1}{4}}[\eta\frac{dF}{d\eta} - 3F] = \frac{\nu}{4^{\frac{1}{4}}}\frac{(Gr_x)^{\frac{1}{4}}}{x}[\eta\frac{dF}{d\eta} - 3F] \tag{9}$$

The partial differential equations of momentum and energy can thus be transformed to ordinary differential equations. Through algebraic manipulation, one may get the following equations:

$$\text{Momentum:}\frac{d^3F}{d\eta^3} + 3F\frac{d^2F}{d\eta^2} - 2(\frac{dF}{d\eta})^2 + \theta = 0 \tag{10}$$

$$\text{Energy:}\frac{d^2\theta}{d\eta^2} + 3PrF\frac{d\theta}{d\eta} = 0 \tag{11}$$

From the above differential equations, it may be observed that F and θ serve as functions of η. The partial differential equations hence have been transformed into ordinary differential equations. the corresponding transformed boundary conditions for momentum can be expressed as:

$$\eta = 0 \qquad F = 0 \qquad \frac{dF}{d\eta} = 0 \tag{12}$$

$$\eta \to \infty \qquad \frac{dF}{d\eta} = 0 \tag{13}$$

Similarly, for energy equation the transformed boundary conditions can be expressed as:

$$\eta = 0 \qquad \theta = 1 \tag{14}$$

$$\eta = \infty \qquad \theta = 0 \tag{15}$$

For some provide Prandtl number, the solutions have to be developed for ordinary differential equations which have been coupled. Pohlhausen conducted numerical solution for Pr=0.733, which later by Schuh, developed equations for solutions at Pr=10, 100 and 1000 Sissom and Pitts (1972). The approximate solutions of Pohlhausen were improved by Ostrach Ostrach (1952), who solved the coupled nonlinear ordinary differential equations numerically for Prandtl number ranging between 0.01 to 1000. The local heat trasfer coefficients can be thus obtained from such solitions. The heat flux $q_w^{"}(x)$ at any aid position x is given as:

$$q_w^{"}(x) = -k(\frac{\partial T}{\partial y})_{y=0} = kcx^{\frac{-1}{4}}(T_w - T_\infty)(\frac{-d\theta}{d\eta})_{\eta=0} \tag{16}$$

where $c = (\frac{g\beta\theta_w}{4\nu^2})^{\frac{1}{4}}$ and the local heat transfer coefficient h(x) is given as:

$$h(x) = \frac{q_w^{"}(x)}{T_w - T_\infty} = -kcx^{\frac{-1}{4}}(\frac{d\theta}{d\eta})_{\eta=0} \tag{17}$$

The localized Nusselt's number Nu_x is given as:

$$Nu_x = \frac{hx}{k} = cx^{\frac{3}{4}}(-\frac{d\theta}{d\eta})_{\eta=0} = (\frac{Gr_x}{4})^{\frac{1}{4}}(-\frac{d\theta}{d\eta})_{\eta=0} \tag{18}$$

or

$$\frac{Nu_x}{(Gr_x)^{\frac{1}{4}}} = \frac{1}{\sqrt{2}}(\frac{-d\theta}{d\eta})_{\eta=0} \tag{19}$$

The Gr_x is the localized Grashof number and it can be expressed as:

$$Gr_x = \frac{g\beta(T_w - T_\infty)x^3}{\nu^2} \tag{20}$$

The local heat flux $g_w^{"}(x)$ be integrated over local length L of the plate in order to obtain the average heat flux (local) per unit width such as:

$$q_w^{-} = \frac{1}{L}q_w^{"}(x)dx = \frac{4}{3}kcL^{\frac{-1}{4}}(T_w - T_\infty)(\frac{-d\theta}{d\eta})_{\eta=0} \tag{21}$$

and the average heat flux $\overline{\mathbf{q}}_w$ is used to define the average heat transfer coefficient $\overline{\mathbf{h}}$

$$\overline{\mathbf{h}} = \frac{\overline{\mathbf{q}}_w}{\overline{\mathbf{T}}_w - \overline{\mathbf{T}}_\infty} = \frac{4}{3}\mathbf{k} \circ \mathbf{L}^{-1/4}\left(-\frac{\mathrm{d}\theta}{\mathrm{d}\eta}\right)_{\eta=0} \tag{22}$$

Finally, the average heat transfer coefficient $\overline{\mathbf{h}}$ is used to obtain the average Nusselt number

$$\overline{\mathrm{Nu}}_L = \frac{\overline{\mathbf{h}}\,\mathbf{L}}{\mathbf{k}} = \frac{\mathbf{L}}{\mathbf{k}}\frac{\overline{\mathbf{q}}_w}{(\overline{\mathbf{T}}_w - \overline{\mathbf{T}})} = \frac{4}{3}\mathbf{c}\,\mathbf{L}^{3/4}\left(-\frac{\mathrm{d}\theta}{\mathrm{d}\eta}\right)_{\eta=0} = \frac{4}{3}\left(\frac{\mathrm{G}_{\mathrm{T_L}}}{4}\right)^{1/4}\left(-\frac{\mathrm{d}\theta}{\mathrm{d}\eta}\right)_{\eta=0} \tag{23}$$

or

$$\overline{\mathrm{Nu}}_L = \frac{4}{3}\mathrm{Nu}_{k=L} \tag{24}$$

where $\mathrm{G}_{\mathrm{T_L}} = \mathrm{g}(\sqrt{\mathbf{T}}_w - \overline{\mathbf{T}}_\infty)\mathbf{L}^3/\nu^2$ is the Grashof number based on the plate length. LeFevre, as reported by Ede Ede (1967) as well as Churchill and Usagi Churchill and Usagi (1972) using asymptotic results at $\mathrm{Pr} = 0$ and $\mathrm{Pr} = \infty$ as a guide, proposed an expression that fits to the numerical results very closely and proposed the following interpolation equation:

$$\mathrm{Nu}_x = \left[\frac{0.75\sqrt{\mathrm{Pr}}}{(2.43478 + 4.884\sqrt{\mathrm{Pr}} + 4.95283\,\mathrm{Pr})^{1/4}}\right](\mathrm{G}_{\mathrm{r_X}})^{1/4} \tag{25}$$

The average Nusselt number is

$$\overline{\mathrm{Nu}}_L = \frac{\overline{\mathbf{h}}\,\mathbf{L}}{\mathbf{k}} = \left[\frac{\sqrt{\mathrm{Pr}}}{(2.43478 + 4.884\sqrt{\mathrm{Pr}} + 4.95283\,\mathrm{Pr})^{1/4}}\right](\mathrm{G}_{\mathrm{r_L}})^{1/4} \tag{26}$$

$$\mathrm{T}_w = \mathrm{const}$$

$$10^6 \leq \mathrm{Ra}_L \leq 10^9$$

$$0 \leq \mathrm{Pr} \leq \infty.$$

It is reported that these equations fit to Ostrach's results within $\pm 0.5\%$ over the entire Prandtl number range.

For air, $(\mathrm{Pr} = 0.72)$, and local and average Nusselt numbers are

$$\mathrm{Nu}_x = 0.3568\,(\mathrm{G}_{\mathrm{r_X}})^{1/4} \tag{27}$$

$$\overline{\mathrm{Nu}}_L = 0.4757\,(\mathrm{G}_{\mathrm{T_L}})^{1/4} \tag{28}$$

The asymptotic values of the average Nusselt numbers for isothermal vertical flat plates are given by Le Fevre, as reported in

$$\overline{Nu}_L = 0.800(Gr_L^{\frac{1}{4}} Pr^{\frac{1}{2}}) \quad Pr \to 0 \tag{29}$$

$$\overline{\mathrm{Nu}}_L = 0.670 \left(\mathrm{G}_{\mathrm{T_L}}^{1/4} \mathrm{Pr}^{\cdot\,1/4}\right) \ \mathrm{Pr} \to \infty. \tag{30}$$

Kays give the asymptotic values of the local Nusselt numbers for isothermal vertical flat plates

$$\mathrm{Nu}_x = 0.600 \left(\mathrm{G}_{\mathrm{T_X}} \mathrm{Pr}^{\cdot\,1/2}\right)^{1/4} \quad \mathrm{Pr} \to 0 \tag{31}$$

$$\mathrm{Nu}_x = 0.503 \left(\mathrm{G}_{\mathrm{T_X}} \mathrm{Pr}\right)^{1/4} \qquad \mathrm{Pr} \to \infty. \tag{32}$$

Practical Example

A 40-cm-high and 20-cm-wide flat plate is placed in a vertical position in air at 1-atm pressure and 17 °C. The back side of the plate is insulated, and the thickness of the plate is negligible. The front surface of the plate is at 237 °C.
(a) Plot the velocity and temperature profiles at the middle of the vertical plate.
(b) Estimate the total heat loss from the plate Onur (2023).

Solution Implementation

The velocity and temperature profiles can be drawn at the middle of vertical plates by numerically solving the momentum and energy ordinary differential equations , numerically such that the MATLAB code is

```
clear;
clc;
% Initial guess and mesh
solinit = bvpinit(linspace(0, 8, 10), [0, 0, 0, 0, 0]);
% Solve the BVP
sol = bvp4c(@NATCONV_BVP, @BC, solinit);
% Extract solution
x = sol.x;
y = sol.y;
% Plot the solution
plot(x, y);
legend('y_1', 'y_2', 'y_3', 'y_4', 'y_5');
xlabel('x');
ylabel('Solution Components');
title('Solution of the BVP');
% Define the differential equations
function f = NATCONV_BVP(x, y)
    Pr = 0.69;
    f = zeros(5, 1);
    f(1) = y(2);
    f(2) = y(3);
    f(3) = -3*y(1)*y(3) + 2*y(2)^2 - y(4);
    f(4) = y(5);
    f(5) = -(3*Pr)*y(1)*y(5);
end
% Grid setup
x_vals = linspace(0.01, 0.4, 50);       % Vertical plate length from 1cm to 40cm
y_vals = linspace(0, 0.02, 100);        % From wall up to 2 cm
[X, Y] = meshgrid(x_vals, y_vals);
% similarity solution
eta = sol.x;
f_prime = sol.y(2, :);
```

```
theta = sol.y(4, :);
% Scaling factor for eta
eta0 = (g * beta * (Tw - Tinf) / nu^2)^(1/4);
% Interpolants for f' and
f_interp = @(eta_val) interp1(eta, f_prime, eta_val, 'linear', 'extrap');
theta_interp = @(eta_val) interp1(eta, theta, eta_val, 'linear', 'extrap');
% Initialization of result matrices
U = zeros(size(X));
T = zeros(size(X));
% Computing of u(x,y) and T(x,y)
for i = 1:size(X, 1)
    for j = 1:size(X, 2)
        x_loc = X(i, j);
        y_loc = Y(i, j);
        eta_loc = eta0 * y_loc / x_loc^(1/4);
        U(i, j) = f_interp(eta_loc) * sqrt(g * beta * (Tw - Tinf) * x_loc);
        T(i, j) = Tinf + theta_interp(eta_loc) * (Tw - Tinf);
    end end
% Plot contour of velocity
figure;
contourf(X, Y, U, 30, 'LineColor', 'none');
colorbar;
xlabel('x (m)');
ylabel('y (m)');
title('Contour of u velocity (m/s)');
% Plot contour of temperature
figure;
contourf(X, Y, T, 30, 'LineColor', 'none');
colorbar;
xlabel('x (m)');
ylabel('y (m)');
title('Contour of temperature (K)');
% Defination of the boundary conditions
function res = BC(ya, yb)
    res = zeros(5, 1);
    res(1) = ya(1);       % y1(0) = 0
    res(2) = ya(2);       % y2(0) = 0
    res(3) = ya(4) - 1;   % y4(0) = 1
    res(4) = yb(2);       % y2() = 0
    res(5) = yb(4);       % y4() = 0
end
% Physical constants based on air at film temperature ~400 K
g = 9.81;                       % m/s^2
beta = 1/400.15;                % 1/K
Tw = 510.15;                    % K
Tinf = 290.15;                  % K
Pr = 0.69;
nu = 2.8e-5;                    % m^2/s, kinematic viscosity
x = 0.2;                        % Vertical location for local similarity solution (mid-height
% Convert eta to physical y
eta_factor = (g * beta * (Tw - Tinf) / nu^2)^(1/4);
y = eta / eta_factor;           % In meters
% Reference velocity
Uref = sqrt(g * beta * (Tw - Tinf) * x);  % In m/s
% Convert to physical quantities
u = f_prime * Uref;                    % Velocity profile (m/s)
T = Tinf + theta * (Tw - Tinf);        % Temperature profile (K)
% Plotting Velocity Profile
```

```
figure;
plot(f_prime, eta, 'b-', 'LineWidth', 2);
xlabel('Dimensionless Velocity, f''(\eta)');
ylabel('\eta');
title('Velocity Profile');
grid on;
% Plot Temperature Profile
figure;
plot(theta, eta, 'r-', 'LineWidth', 2);
xlabel('Dimensionless Temperature, \theta(\eta)');
ylabel('\eta');
title('Temperature Profile');
grid on;
% Plot u(y)
figure;
plot(y, u, 'k', 'LineWidth', 1.5);
xlabel('y (m)');
ylabel('u (m/s)');
title('(a) Variation of u velocity with y');
grid on;
% Plot T(y)
figure;
plot(y, T, 'k', 'LineWidth', 1.5);
xlabel('y (m)');
ylabel('T (K)');
title('(b) Variation of temperature T with y');
grid on;
```

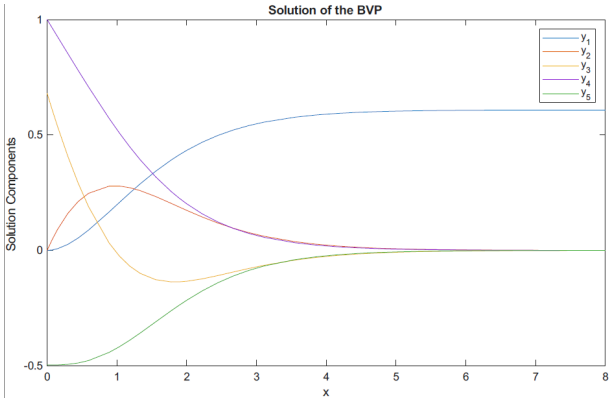


Figure 1: Solution of Momentum and Energy Equations subjected to Boundary Conditions

Through the numerical solution, the velocity and temperature profiles have been computed above, The similarity solutions highlight that the temperature and velocity distributions are given by following expressions:

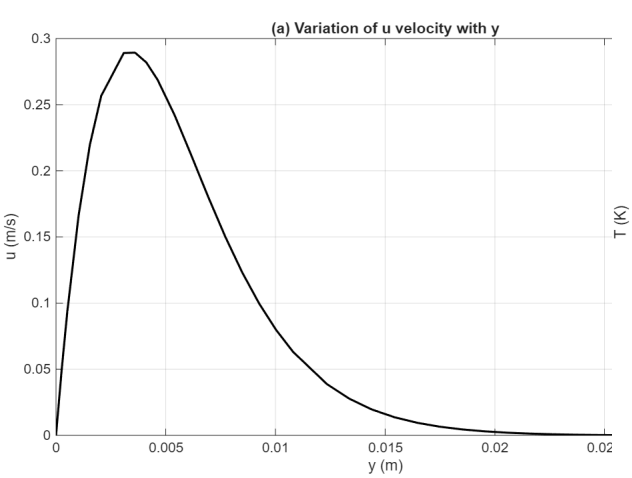
$$u = (\frac{2\nu}{x})\sqrt{Gr_x}\frac{dF}{d\eta}$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}$$

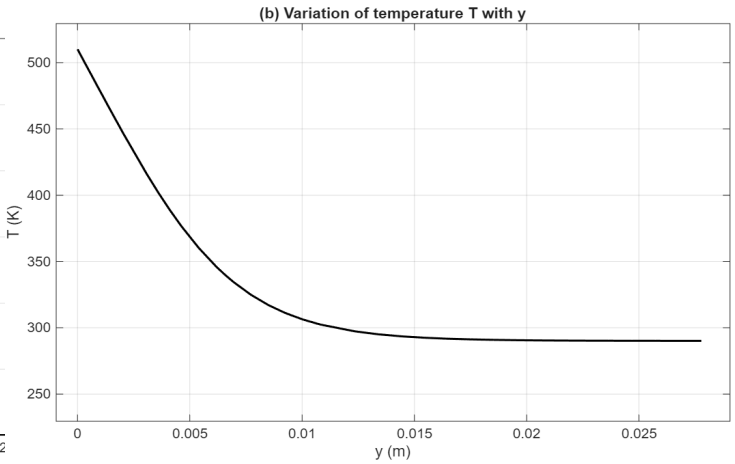
The dimensionless parameters η and Gr_x are given by following expressions:

$$\eta = \frac{y}{x}(\frac{Gr_x}{4})^{\frac{1}{4}}$$

$$Gr_x = \frac{g\beta(T_w - T_\infty)x^3}{\nu^2}$$

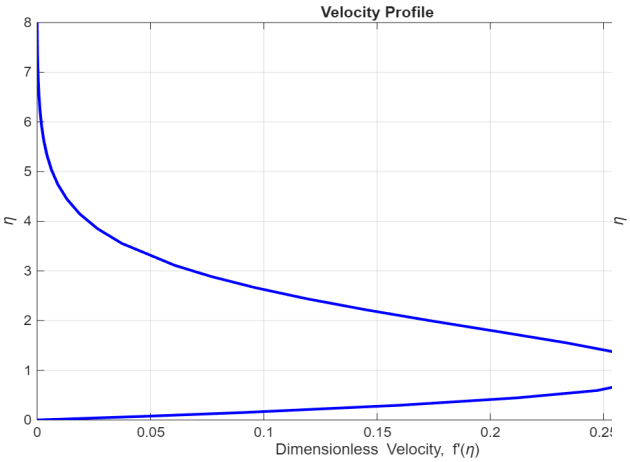


(a) Variation in Velocity u with Vertical Distance y

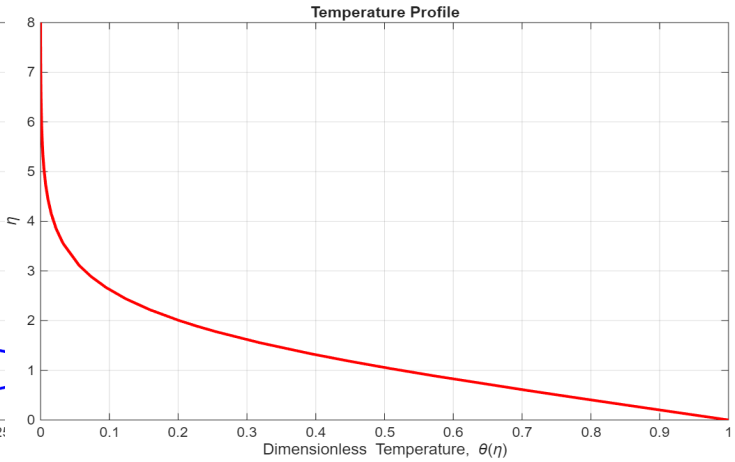


(b) Variation in Temperature Distribution T with respect to Vertical Distance y

Figure 2: Velocity and Temperature Plots with respect to Vertical Distance y



(a) Variation in Velocity u with Nondimensionalized Velocity f'(η)



(b) Variation in Temperature Distribution T with respect to Nondimensionalized Temperature $\theta(\eta)$

Figure 3: Velocity and Temperature Plots with respect to Nondimensionalized Parameters

The air properties are evaluated at the said film temperature. The film temperature is $T_f = \frac{T_2+T_\infty}{2} = 127^oC = 400K$. For temperature of 400 K, $\nu = 26.41 \times 10^{-3} m^2/s$ and $k = 33.8 \times 10^{-3} W/mK$ and Pr=0.69

$$\beta = \frac{1}{T_f} = \frac{1}{400} = 2.5 \times 10^{-3} K^{-1}$$

The dimensionless distance η at the middle of plate (x=0.20 m) can be calculated as:

$$\eta = \frac{y}{x}(\frac{Gr_x}{4})^{\frac{1}{4}} = \frac{y}{x}(\frac{1}{4}\frac{g\beta(T_w - T_\infty)x^3}{\nu^2})^{\frac{1}{4}}$$

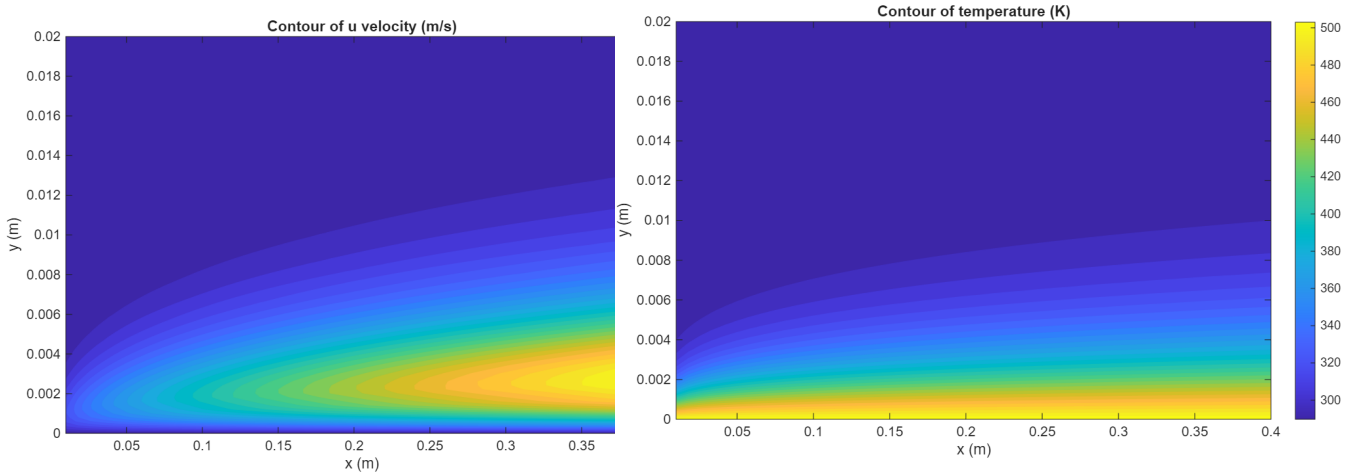
$$\eta = \frac{y}{0.2}(\frac{1}{4}\frac{9.81 \times 2.5 \times 10^{-3}(510-290) \times (0.2)^3}{(26.41 \times 10^{-6})^2})^{\frac{1}{4}}$$

$$y = \frac{\eta}{313.5820731}(m)$$

This equation is rearranged to express y in terms of η.

$$y = \frac{\eta}{313.5820731}\,(m)$$

An equation for the velocity u is determined as



(a) Variation in Velocity u with distance

(b) Variation in Temperature Distribution T with respect to distance

Figure 4: Velocity and Temperature Contours with respect to distances

$$u = \left(\frac{2\nu}{x}\right)\sqrt{Gr_x}\frac{dF}{d\eta} = \left(\frac{2\nu}{x}\right)\sqrt{\frac{g\beta(T_w - T_\infty)x^3}{\nu^2}}\left(\frac{dF}{d\eta}\right)$$

$$u = \left(\frac{2 \times 26.41 \times 10^{-6}}{0.2}\right)\sqrt{\frac{9.81 \times 2.5 \times 10^{-3} \times (510 - 290) \times (0.2)^3}{(26.41 \times 10^{-6})^2}}\left(\frac{dF}{d\eta}\right)$$

$$u = 2.077594764\left(\frac{dF}{d\eta}\right)\ (m/s)$$

Temperature distribution is determined as follows

$$\theta = \frac{T - 290}{510 - 290} \Rightarrow T = 290 + (510 - 290)\theta$$

$$T = 290 + (510 - 290)\theta$$

Since the dimensionless distance η is known, the velocity u and the temperature T can be computed from these equations as function of y.

(b)

One may compute the heat transfer from the plate.

$$\mathrm{Gr}_L = \frac{g\beta(T_w - T_\infty)L^3}{\nu^2} = \frac{9.81 \times 2.5 \times 10^{-3}(510 - 290)(0.4)^3}{(26.41 \times 10^{-6})^2} = 4.95 \times 10^8$$

Flow is laminar since $\mathrm{Gr}_L < 10^9$. The average Nusselt number is given as:

$$\overline{\mathrm{Nu}}_L = 0.4757\mathrm{Gr}_L^{1/4} = 0.4757(4.95 \times 10^8)^{1/4} \approx 70.95$$

$$\overline{h} = \frac{k}{L}\overline{\mathrm{Nu}}_L = \left(\frac{33.8 \times 10^{-3}}{0.4}\right)(70.95) \approx 5.99\, W/m^2K$$

$$q = \overline{h}A(T_w - T_\infty) = (5.99)(0.4 \times 0.20)(510 - 290) = 105.52W$$

Table 1: Numerical Solution Data Table

η	F	F'	F''	θ	θ'
0	0	0	0.6803	1.0000	0.4969
0.2500	0.0187	0.1402	0.4470	0.8759	0.4952
0.5000	0.0656	0.2267	0.2515	0.7531	0.4850
0.7500	0.1284	0.2695	0.0981	0.6345	0.4614
1.0000	0.1976	0.2793	0.0126	0.5235	0.4242
1.2500	0.2662	0.2665	0.0838	0.4233	0.3761
1.5000	0.3297	0.2401	0.1223	0.3360	0.3223
1.7500	0.3857	0.2073	0.1364	0.2622	0.2677
2.0000	0.4333	0.1733	0.1341	0.2018	0.2165
2.2500	0.4725	0.1411	0.1222	0.1535	0.1712
2.5000	0.5041	0.1126	0.1057	0.1156	0.1330
2.7500	0.5292	0.0883	0.0882	0.0864	0.1017
3.0000	0.5487	0.0684	0.0716	0.0642	0.0770
3.2500	0.5637	0.0524	0.0569	0.0475	0.0577
3.5000	0.5751	0.0397	0.0445	0.0350	0.0430
3.7500	0.5838	0.0299	0.0344	0.0257	0.0318
4.0000	0.5903	0.0224	0.0263	0.0188	0.0235
4.2500	0.5951	0.0166	0.0199	0.0138	0.0173
4.5000	0.5987	0.0123	0.0150	0.0100	0.0127
4.7500	0.6014	0.0091	0.0112	0.0073	0.0093
5.0	0.6033	0.0068	0.0066	0.0084	0.0053

Conclusion

The similarity solution for laminar free convection over a vertical flat plate with constant wall temperature has been successfully implemented. The governing equations were transformed into a pair of ordinary differential equations for momentum and energy, which were then solved numerically using appropriate boundary conditions. The analysis for a specific plate in air provided detailed velocity and temperature profiles at the mid-height of the plate. The heat transfer calculation, based on the average Nusselt number correlation, yielded a heat loss of 105.52 W. The results are in agreement with the asymptotic limits for small and large Prandtl numbers as proposed by Le Fevre and Kays, respectively. This study confirms the robustness of the similarity approach in resolving natural convection boundary layer problems and provides a framework for similar heat transfer analyses.

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