Heat Transfer at Constant Rod Example

Muhammad Abdullah Zafar Ghauri

26th February 2025

Abstract

Heat Transfer at a Constant Rod is evaluated by means of MATLAB by varying specific perimeters through Finite Difference Method subjected to boundary conditions.

1 Introduction

The heat transfer equation is generally represented as:

$$\frac{\partial T}{\partial t} = \alpha^2 \frac{\partial^2 T}{\partial z} \tag{1}$$

The above heat transfer equation can be expressed in terms of finite difference technique which shall result it in the following form:

$$\frac{T_i^{j+1} - T_i^j}{\Delta T} = \alpha(\frac{T_{i+1}^{j+1} - 2T_i^{j+1} + T_{i-1}^{j+1}}{\Delta z^2}) \tag{2}$$

The following system can be expressed in terms of finite-difference as

$$T_i^{j+1} - T_i^j = \frac{\alpha \Delta t}{\Delta z^2}(T_{i+1}^{j+1} - 2T_j^{j+1} + T_{i-1}^{j-1}) \forall s = \frac{\alpha \Delta t}{\Delta z^2} \tag{3}$$

$$T_i^j = T_i^{j+1} - s(T_i^{j+1} + 1 - 2T_j^{j+1} + T_{i-1}^{j-1}) \tag{4}$$

Where j is index used for time

$$T_i^j = -sT_{i+1}^{j+1} + (1+2s)T_i^{j+1} - sT_{i+1}^{j+1} \tag{5}$$

where i is used as an index for position The associated boundary conditions to the following finite difference equation are as Initial conditions are as follows

$$t = 0; T = T_o \tag{6}$$

The assosiated boundary conditions are as follows:

$$z = 0; T = T_s \tag{7}$$

$$z = L; T = T_o \tag{8}$$

Further elaborating the initial conditions and boundary conditions, they have been assigned with numerical values with α=0.0005; L=0.25; T_f=20 (which is the time constraint). The consequent initial and boundary conditions can be expressed as:

$$t = 0; T = 28^oC \tag{9}$$

$$z = 0; T = 300^oC \tag{10}$$

$$z = L; T = 28^oC \tag{11}$$

```
    clc;
alpha = 0.0005;
L=0.25;
tf=20; %Time Limit
T0=28; %Ambinet Temperature
Ts=300; %Heat Source Temperature
%Step
Nt=101;
Nz=151;
t=linspace (0,tf,Nt);
z=linspace(0,L,Nz);
dt=t(2)-t(1);
dz=z(2)-z(1);
%Preparation of Matrix
A=zeros(Nz,Nz); %position Matrix
B=zeros (Nz,1); % Constant
T=zeros(Nt,Nz);
%Applicaction of Initial COnditions
T(1,:)=ones(1,Nz)*T0;
%BC-1
A(1,1)=1;
B(1)=Ts;
%BC-2
A(end,end)=1;
B(end)=T0;
%For calculation simplification
s=alpha*dt./dz.^2;
%Formation of tridiagonal matrix
for j=2:Nt %FOR loop for time start at 2nd iteration, j=time
for i=2:Nz-1  %i=position
    A(i,i-1)=-s;
    A(i,i)=1+2*s;
    A(i,i+1)=-s;
    B(i)=T(j-1,i); %j-1 means 2-1=1
end
T(j,:)=A\B;
figure(1)
plot(z,T(j,:), 'LineWidth',3)
title(['Temperature at t=', num2str(t(j)), 'sec'])
axis([0 L 0 Ts])
xlabel('Distance')
ylabel('Temperature')
pause (0.01)
end
figure(2)
```

```
imagesc (z,t,T)
title('Distribution Temperature')
xlabel('Distance')
ylabel('Time')
colormap jet
colorbar
grid on
```

Listing 1: Heat Transfer through Constant Rod

The subsequent temperature profiles were observed as: Varying the values,

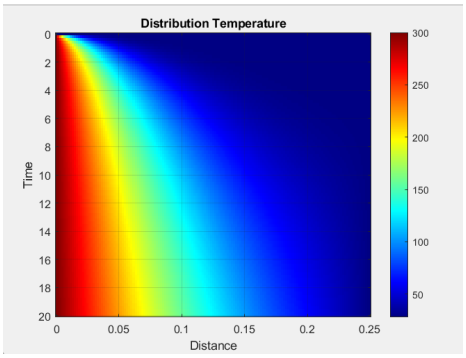


Figure 1: Temperature Distribution Contour about the Cross Sectional Length of Rod

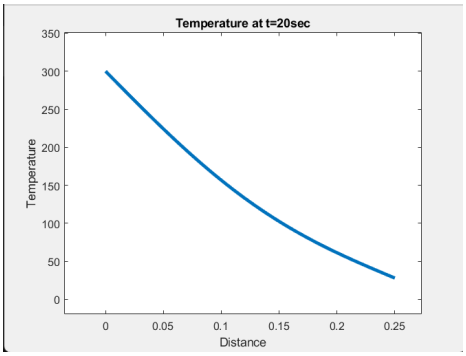


Figure 2: Varying Temperature Profile

such that α=0.100 and the initial conditions increased to

$$t = 0; T = 100^o C \tag{12}$$

$$z = 0; T = 400^o C \tag{13}$$

$$z = L; T = 100^o C \tag{14}$$

and the time delay was increased to 200 s, the temperature variation profile became more linear.

```
    clc;
alpha = 0.100;
L=0.25;
tf=200; %Time Limit
T0=100; %Ambinet Temperature
Ts=400; %Heat Source Temperature
%Step
Nt=101;
Nz=151;
t=linspace (0,tf,Nt);
z=linspace(0,L,Nz);
dt=t(2)-t(1);
dz=z(2)-z(1);
%Preparation of Matrix
A=zeros(Nz,Nz); %position Matrix
B=zeros (Nz,1); % Constant
T=zeros(Nt,Nz);
%Applicaction of Initial COnditions
T(1,:)=ones(1,Nz)*T0;
%BC-1
A(1,1)=1;
B(1)=Ts;
%BC-2
A(end,end)=1;
B(end)=T0;
%For calculation simplification
s=alpha*dt./dz.^2;
%Formation of tridiagonal matrix
for j=2:Nt %FOR loop for time start at 2nd iteration, j=time
for i=2:Nz-1  %i=position
    A(i,i-1)=-s;
    A(i,i)=1+2*s;
    A(i,i+1)=-s;
    B(i)=T(j-1,i); %j-1 means 2-1=1
end
T(j,:)=A\B;
figure(1)
plot(z,T(j,:), 'LineWidth',3)
title(['Temperature at t=', num2str(t(j)), 'sec'])
axis([0 L 0 Ts])
xlabel('Distance')
ylabel('Temperature')
pause (0.01)
end
figure(2)
imagesc (z,t,T)
title('Distribution Temperature')
xlabel('Distance')
ylabel('Time')
colormap jet
colorbar
grid on
```

Listing 2: Heat Transfer through Constant Rod for Elevated temperatures and

time delay

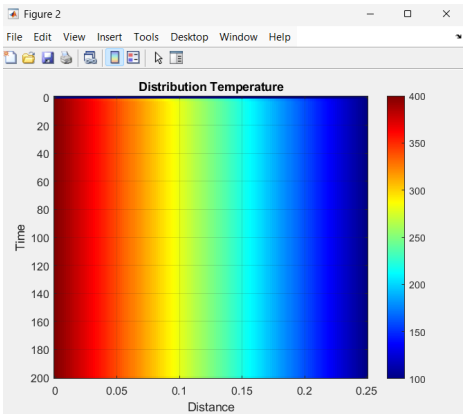


Figure 3: Temperature Distribution Contour with improved boundary and initial conditions

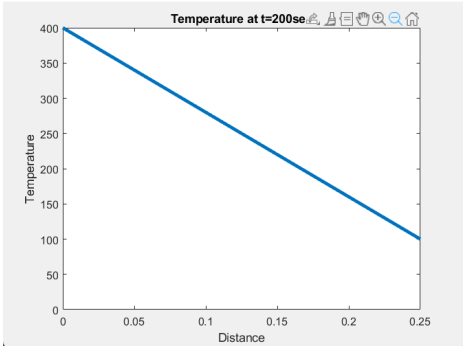


Figure 4: Variation in Temperature Profile by increasing time delay

2 Conclusion

The numerical analysis of heat transfer along a constant rod using the finite difference method in MATLAB provided valuable insights into the thermal behavior under varying conditions. The results demonstrated that temperature distribution along the rod followed a predictable trend, with heat gradually diffusing from the high-temperature boundary towards the lower-temperature regions. Increasing the thermal diffusivity (α) led to a more rapid temperature stabilization, confirming that materials with higher conductivity facilitate faster heat propagation. Additionally, modifications in boundary conditions

significantly influenced the temperature gradients, highlighting the role of thermal insulation and heat source placement in practical applications. Extending the simulation time revealed that the temperature profile became more linear over time, indicating a steady-state condition as thermal equilibrium was approached. These findings validate the finite difference method as a reliable tool for modeling transient heat conduction and emphasize its potential for further studies involving complex geometries, non-uniform heat sources, and varying material properties.