Heat Transfer at Constant Rod Example

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Abstract

Heat Transfer at a Constant Rod is evaluated by means of MATLAB by varying specific perimeters through Finite Difference Method subjected to boundary conditions.

1 Introduction

The heat transfer equation is generally represented as:

$$\frac{\partial T}{\partial t} = \alpha^2 \frac{\partial^2 T}{\partial z} \tag{1}$$

The above heat transfer equation can be expressed in terms of finite difference technique which shall result it in the following form:

$$\frac{T_i^{j+1} - T_i^j}{\Delta T} = \alpha(\frac{T_{i+1}^{j+1} - 2T_i^{j+1} + T_{i-1}^{j+1}}{\Delta z^2}) \tag{2}$$

The following system can be expressed in terms of finite-difference as

$$T_i^{j+1} - T_i^j = \frac{\alpha \Delta t}{\Delta z^2}(T_{i+1}^{j+1} - 2T_j^{j+1} + T_{i-1}^{j-1}) \forall s = \frac{\alpha \Delta t}{\Delta z^2} \tag{3}$$

$$T_i^j = T_i^{j+1} - s(T_i^{j+1} + 1 - 2T_j^{j+1} + T_{i-1}^{j-1}) \tag{4}$$

Where j is index used for time

$$T_i^j = -sT_{i+1}^{j+1} + (1+2s)T_i^{j+1} - sT_{i+1}^{j+1} \tag{5}$$

where i is used as an index for position The associated boundary conditions to the following finite difference equation are as Initial conditions are as follows

$$t = 0; T = T = sin(\pi z) \tag{6}$$

The associated boundary conditions are as follows:

$$z = 0; T = 0^o C \tag{7}$$

$$z = L; T = 0^oC \tag{8}$$

Further elaborating the initial conditions and boundary conditions, they have been assigned with numerical values with α=1.0; L=1.0; T_f=0.20 (which is the time constraint). The consequent initial and boundary conditions can be expressed as:

$$t = 0; T = sin(\pi z) \tag{9}$$

$$z = 0; T = 0^oC \tag{10}$$

$$z = L; T = 0^oC \tag{11}$$

```
  clc;
alpha =1;
L=1;
tf=.200; %Time Limit
%Step
Nt=201;
Nz=201;
t=linspace (0,tf,Nt);
z=linspace(0,L,Nz);
dt=t(2)-t(1);
dz=z(2)-z(1);
%Preparation of Matrix
A=zeros(Nz,Nz); %position Matrix
B=zeros (Nz,1); % Constant
T=zeros(Nt,Nz);
%Applicaction of Initial COnditions
T(1,:)=ones(1,Nz).*sin(pi.*z);
%BC-1
A(1,1)=1;
B(1)=0;
%BC-2
A(end,end)=1;
B(end)=0;
%For calculation simplification
s=alpha*dt./dz.^2;
%Formation of tridiagonal matrix
for j=2:Nt %FOR loop for time start at 2nd iteration, j=time
for i=2:Nz-1  %i=position
    A(i,i-1)=-s;
    A(i,i)=1+2*s;
    A(i,i+1)=-s;
    B(i)=T(j-1,i); %j-1 means 2-1=1
end
T(j,:)=A\B;
figure(1)
plot(z,T(j,:), 'LineWidth',3)
title(['Temperature at t=', num2str(t(j)), 'sec'])
axis([0 L 0 1])
xlabel('Distance')
ylabel('Temperature')
pause (0.001)
end
figure(2)
imagesc (z,t,T)
title('Distribution Temperature')
```

```
xlabel('Distance')
ylabel('Time')
colormap jet
colorbar
grid on
```

Listing 1: Heat Transfer through Constant Rod

The subsequent temperature profiles were observed as:

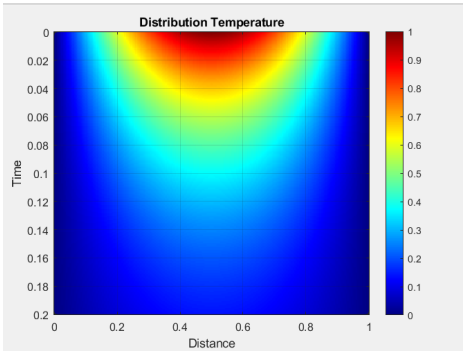


Figure 1: Temperature Distribution Contour about the Cross Sectional Length of Rod

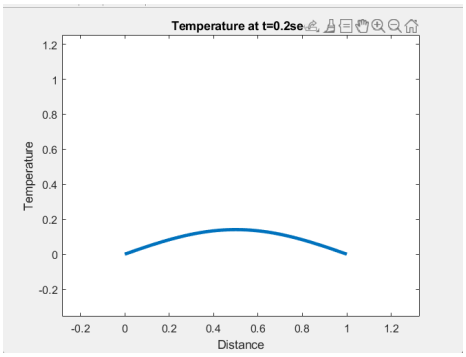


Figure 2: Varying Temperature Profile

Varying the values, such that α=2(sinπcosπ) and the initial conditions increased to

$$t = 0; T = sin(3\pi z) - 2cos(3\pi z) \tag{12}$$

$$z = 0; T = 0^oC \tag{13}$$

$$z = L; T = 0^oC \tag{14}$$

and the time delay was increased to 0.400 s, the temperature variation profile became more linear.

```
clc;
alpha =2*(sin(pi)*cos(pi));
L=1;
tf=.400; \%Time Limit
\%Step
Nt=201;
Nz=201;
t=linspace (0,tf,Nt);
z=linspace(0,L,Nz);
dt=t(2)-t(1);dz=z(2)-z(1);
\%Preparation of Matrix
A=zeros(Nz,Nz); \%position Matrix
B=zeros (Nz,1);
\% Constant
T=zeros(Nt,Nz);
\%Applicaction of Initial Conditions
T(1,:)=ones(1,Nz).*sin(3*pi.*z)-2*cos(3*pi.*z);
\%BC-1
A(1,1)=1;B(1)=0;
\%BC-2
A(end,end)=1;
B(end)=0;
\%For calculation simplifications=alpha*dt./dz.\^2;
\%Formation of tridiagonal matrix
for j=2:Nt \%FOR loop for time start at 2nd iteration, j=time
for i=2:Nz-1 \%i=position
A(i,i-1)=-s;A(i,i)=1+2*s;
A(i,i+1)=-s;B(i)=T(j-1,i); \%j-1 means 2-1=1
end
T(j,:)=A\B;
figure(1)
plot(z,T(j,:), 'LineWidth',3)title(['Temperature at t=', num2str
 (t(j)), 'sec'])
axis([0 L 0 1])
xlabel('Distance')
ylabel('Temperature')
pause (0.001)
end
figure(2)
imagesc (z,t,T)
title('Distribution Temperature')
xlabel('Distance')
ylabel('Time')
colormap jet
colorbar
grid on
```

Listing 2: Heat Transfer through Constant Rod for Elevated temperatures and time delay

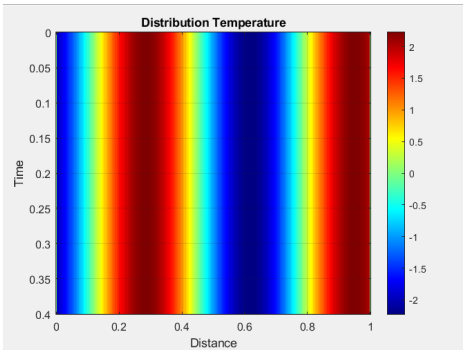


Figure 3: Temperature Distribution Contour with improved boundary and initial conditions

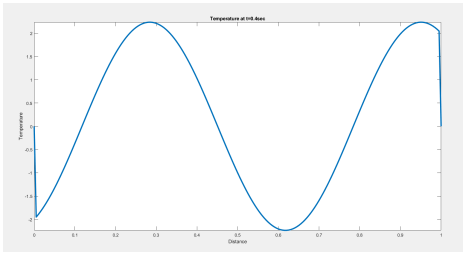


Figure 4: Variation in Temperature Profile by increasing time delay

2 Conclusion

The numerical analysis of heat transfer in a constant rod using the finite difference method revealed significant variations in temperature distribution under different conditions. In the first case, with α=1.0 and an initial temperature profile of T=sin(πz), the temperature evolution followed a smooth distribution, gradually decreasing over time while maintaining a symmetric profile. In contrast, the second case, where α was modified to 2(sinπcosπ) and the initial condition was changed to T=sin(3πz)2cos(3πz), exhibited a more complex initial temperature distribution. The increased thermal diffusivity and the modified initial conditions resulted in a steeper temperature gradient, leading to a more linearized heat dissipation over time. Additionally, the extended time limit in the second case provided a clearer visualization of the gradual stabilization of temperature across the rod. This comparison highlights the sensitivity of heat transfer characteristics to initial and boundary conditions, as well as the impact of thermal diffusivity on the rate and pattern of temperature distribution. The results confirm that adjusting these parameters significantly influences the transient behavior of heat conduction, demonstrating the effectiveness of the finite difference method in capturing these variations.