Empirical Correlations for Free Convection: Vertical Cylinder : Analysis using Uniform Wall Temperature (UWT) and Uniform Wall Heat Flux (UHF)

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Abstract

Free convection from heated vertical cylindrical surfaces is an important phenomenon in thermal-fluid sciences, yet remains challenging to model analytically due to the nonlinear coupling of momentum and energy transport. This study presents a strictured analytical and numerical approach to solution of problems using empirical correlations governing free convection over a vertical cylinder corresponding to both Uniform Wall Temperature (UWT) and Uniform Wall Heat Flux (UHF) boundary conditions. By using dimensionless groups—Rayleigh (Ra), Grashof (Gr), Prandtl (Pr), and Nusselt (Nu) numbers—this work synthesizes existing experimental data into design-relevant expressions for both laminar and turbulent regimes. MATLAB-based simulations evaluate local and average heat transfer behaviors iteratively, employing updated thermophysical properties at film temperatures. Visualizations include wall temperature profiles, local Nu and Gr distributions, and 2D/3D temperature fields. These insights not only confirm the validity of classic correlations but also demonstrate the importance of accurate thermophysical estimation across iterative cycles. The dataset and accompanying code offer an adaptable foundation for further convection modeling in design, education, and applied research.

Keywords: Free convection, Empirical correlations, Nusselt number, Rayleigh number, MATLAB simulation, Uniform wall temperature, Heat transfer, Vertical cylinder

Free convection is a very complex phenomenon and the analysis relies much upon the experimental data in order to develop effective heat transfer relations. The Prandtl number and Grashof number are key dimensionless numbers in free convection analysis and the correlations develop are functions of said dimensionless numbers. Below are some empirical correlations for both laminar and turbulent free convection with uniform wall temperature (UWT) and uniform heat flux (UHF) boundary conditions. The experimental studies indicate that the averaged localized Nusselt's number for free convection can be represented as:

$$\overline{Nu_L} = C(Gr_L Pr)^m = CRa_L^m \tag{1}$$

Where Ra_L is the localized Rayleigh number and is defined as the product of localized Grashof number and Prandtl number, mathematically as

$$Ra_L = Gr_L Pr = \frac{g\beta(T_w - T_\infty)L^3}{\nu^2} Pr = \frac{g\beta(T_w - T_\infty)L^3}{\nu^2 \alpha}$$

L is the characteristic length, while the values of C and m in 1 are dependent on surface geometry, and the flow characteristics whether it is laminar or turbulent. The fluidic properties are evaluated using the film temperature T_f as:

$$T_f = \frac{T_w + T_\infty}{2}$$

T_w and T_∞ are surface and ambient temperatures. Considering free convection from vertical plate, the average convective heat transfer coefficient is given as:

$$\bar{h} = \frac{1}{L}\int_o^L h(x)dx$$

Where $h(x)$ is the local convective heat transfer coefficient along the vertical hot plate. Assuming the local heat transfer coefficient being dependent on x in the following form:

$$h = Cx^{-n}$$

The averaged heat transfer hence can be evaluated as:

$$\bar{h} = \frac{1}{L}\int_o^L Cx^{-n}dx = \frac{C}{1-n}L^{-n} = (\frac{1}{1-n}h(x))_{x=L}$$

The average Nusselt number ultimately takes shape as:

$$\overline{Nu_L} = (\frac{1}{1-n}Nu_x)_{x=L} \tag{2}$$

Where $Nu_{x=L}$ is the local Nusselt number evaluated at x=L. The characteristic dimension L used in Grashof number Gr_L and Nusselt number $\overline{Nu_L}$ is dependent on the geometry concerned. Considering a vertical plate of height L and for horizontal cylinder, it is the cylinder diameter D.
In many engineering applications, the natural convection heat transfer from surface of cylinder of some cross sectional area has important applications in many engineering applications, especially in heating and cooling equipment.,

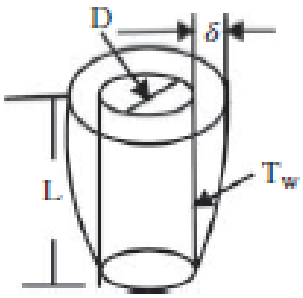


Figure 1: Vertical cylinder in free convection Onur (2023)

Uniform Wall Temperature (UWT)

The equations of continuity, momentum, and energy can be written for vertical cylinder. These equations are later reduced to boundary layer formulations through magnitude analysis. Consider an isothermal vertical cylinder, similar to one given as 1. If the boundary layer thickness δ is less than the cylinder diameter D (such that $\delta << D$), the vertical isothermal cylinder can be modeled as a vertical flat, as discussed in Sparrow and Gregg (1956) . The general criterion is:

$$\frac{D}{L} \geq \frac{35}{Gr_L^{\frac{1}{4}}}Pr = 0.72 \tag{3}$$

Where

$$Gr_L = \frac{g\beta(T_w - T_\infty)L^3}{\nu^2}$$

is the Grashof number based on the height of cylinder, D is the cylinder diameter and L is the length of cylinder. 3 is called thick cylinder limit and was derived by Sparrow and Gregg (1956). Provided the criterion is satisfied, one can use the vertical plate relations, which can be used for estimating the average Nusselt number for vertical cylinders with 5% accuracy as discussed by Sparrow and Gregg (1956). From available literature, the correlations concerning free convection from slender vertical cylinders show widespread, and one should consider to use the correct length in Grashof number, which is the vertical length of cylinder.
For typically small D/L ratios, the Nusselt number can be enhanced by the effect of cylinder curvature. Nellis and Klein (2021) recommends the following correlation:

$$\overline{Nu_L} = \frac{\bar{h}L}{k} = \overline{Nu_{L,nc}}\frac{\xi}{ln(1+\xi)} \tag{4}$$

Where $\overline{Nu_{L,nc}}$ is the average Nusselt number evaluated using vertical plate correlation and ξ is given by:

$$\xi = \frac{1.8}{\overline{Nu_{L,nc}}}\frac{L}{D}$$

Arpaci et al. (2000) recommended the following correlation for isothermal vertical cylinder:

$$\overline{Nu_L} = C\pi_N^n \tag{5}$$

where

$$\pi_N = \frac{Ra_L}{1 + \frac{C_o}{Pr}}$$

$$Ra_L = \frac{g\beta(T_w - T_\infty)L^3}{\alpha\nu}$$

Where $C_o, C,$ and n are evaluated using 1. Fluidic properties are evaluated at film temperature T_f, given as $T_f = \frac{T_w + T_\infty}{2}$, where T_w and T_∞ are cylinder surface and free stream temperatures respectively. L is the height of cylinder in the gravitational direction.

	C_0	C	n
$10^4 \leq \pi_N \leq 10^9$	0.492	0.67	0.25
$10^9 \leq \pi_N \leq 10^{12}$	0.492	0.15	0.33

Table 1: Constant Table

Gebhart (1961) recommended the following correlation for isothermal vertical cylinders for both laminar regime **Laminar:**

$$\overline{Nu} = \frac{\bar{h}L}{k} = 0.021(Gr_L Pr)^{0.4} \tag{6}$$

$$Gr_L \leq 10^9$$

$$\frac{D}{L} \geq \frac{35}{Gr_L^{\frac{1}{4}} Pr} = 0.72$$

Turbulent:

$$\overline{Nu} = \frac{\bar{h}L}{k} = 0.55(Gr_L Pr)^{0.25} \tag{7}$$

$$Gr_L \geq 10^9$$

$$\frac{D}{L} \geq \frac{35}{Gr_L^{\frac{1}{4}} Pr} = 0.72$$

Where

$$Gr_L = \frac{g\beta(T_w - T_\infty)L^3}{\nu^2}$$

is the Grashof number based on the height of cylinder and fluidic properties are evaluated at film temperature T_f, given as $T_f = \frac{T_w + T_\infty}{2}$, where T_w and T_∞ are cylinder surface and free stream temperatures respectively. proposed the following correlation for free convection from an isothermal vertical cylinder for laminar region:

$$\sqrt{\overline{Nu_L}} = \left[0.6\sqrt{\left(\frac{L}{D}\right)} + 0.387\left[\frac{Ra_L}{\left[1 + \left(\frac{0.492}{Pr}\right)^{\frac{9}{16}}\right]^{\frac{16}{9}}}\right]^{\frac{1}{6}}\right] \tag{8}$$

$$\overline{Nu} = \frac{\bar{h}L}{k}$$

$$Ra_L < 10^9$$

$$T_w = \text{constant}$$

Where

$$Gr_L = \frac{g\beta(T_w - T_\infty)L^3}{\nu^2}$$

is the Grashof number and $Ra_L = Gr_L \times Pr$ is the Rayleigh number based on cylinder height.

Uniform Heat Flux

For vertical cylinder subjected to uniform heat flux, reports the following correlations based on experimental work:

$$\overline{Nu_D} = \frac{\bar{h}L}{k} = 0.6\left(Ra_D\frac{D}{L}\right)^{0.25} \tag{9}$$

$$Ra_D\frac{D}{L} \geq 10^4$$

$$\overline{Nu_D} = \frac{\bar{h}L}{k} = 1.37\left(Ra_D\frac{D}{L}\right)^{0.16} \tag{10}$$

$$0.05 \leq Ra_D\frac{D}{L} \leq 10^4$$

$$\overline{Nu_D} = \frac{\bar{h}L}{k} = 0.93\left(Ra_D\frac{D}{L}\right)^{0.05} \tag{11}$$

$$Ra_D\frac{D}{L} \leq 0.05$$

Where D is the diameter of cylinder and is dependent on the diameter D.

$$Ra_L = \frac{g\beta(T_w - T_\infty)L^3}{\alpha\nu}$$

The thermal conductivity factor β is evaluated at the film temperature T_f which is defined as $T_f = \frac{T_w+T_\infty}{2}$, where T_w and T_∞ are cylinder surface and free stream temperatures respectively. It is typically concerned at $\beta = \frac{1}{T_\infty}$. For isothermal vertical cylinder subjected to UHF, proposed a correlation as:

$$\overline{Nu_D} = \frac{\bar{h}D}{k} = 0.36 + \frac{0.67\left(Ra_D\frac{D}{L}\right)^{\frac{1}{4}}}{\left[1+\left(\frac{0.492}{Pr}\right)^{\frac{9}{16}}\right]^{\frac{4}{9}}} \tag{12}$$

Where

$$Gr_D = \frac{g\beta(T_w - T_\infty)D^3}{\nu^2}$$

is the Grashof number on the basis of cylindrical diameter and $Ra_D = Gr_D \times Pr$ is the Rayleigh number based on cylinder height.

Practical Example

A 20-cm-diameter, 1-m-long vertical cylinder is in quiescent air at 1-atm pressure and 27°C. If the cylinder has a surface temperature of 70 °CC, estimate the heat loss by free convection from the cylinder.Onur (2023).

Solution

$$T_w = 70^o\ C = 343\ \text{K}$$

Mean temperature:

$$T_\infty = 27^oC = 300\ \text{K}$$

The film temperature T_f can be evaluated as:

$$T_f = \frac{T_w + T_\infty}{2} = \frac{343 + 300}{2}$$

$$T_f = 321.5\ K$$

Air properties at 321.5 K:

$$\nu = 18 \times 10^{-6}\ \text{m}^2/\text{s}, \quad k = 0.0278\ \text{W/mK}, \quad Pr = 0.703$$

The thermal coefficient β is approximately equal to the reciprocal of film temperature. So

$$\beta = \frac{1}{T_f} = \frac{1}{321.5}$$

$$\beta = 0.00311\ K^{-1}$$

Through the Sparrow and Gregg (1956) criterion, it has to be evaluated, given as

$$\frac{D}{L} \geq \frac{35}{Gr_L^{\frac{1}{4}}} Pr = 0.72$$

The Grashof number at cylindrical diameter D can be calculated as

$$Gr_L = \frac{g\beta(T_w - T_\infty)L^3}{\nu^2} = Gr_L = \frac{9.81 \times (0.00311) \times (343 - 300) \times (1)^3}{(20.92 \times 10^{-6})^2} \approx 4 \times 10^9$$

Now,

$$\frac{D}{L} = \frac{0.2}{1} = 0.2$$

$$\frac{D}{L} \geq \frac{35}{Gr_L^{\frac{1}{4}}} Pr = 0.72 \rightarrow 0.2 \geq \frac{35}{(4 \times 10^9)^{\frac{1}{4}}} = 0.138$$

Now

$$0.2 > 0.138$$

Thus by Sparrow and Gregg (1956) criterion, the curvature of cylinder can be ignored and it can be treated as a vertical wall. The characteristic length of wall is 1 m. Since the flow is turbulent, as $Gr_L > 1 \times 10^9$, so the Churchill and Chu (1975) correlation for turbulent flow can be used for calculating the averaged Nusselt number

$$(\overline{\mathrm{Nu}}_L) = 0.825 + \frac{0.387 Ra_L^{\frac{1}{6}}}{\left[1 + \left(\frac{0.492}{\mathrm{Pr}}\right)^{9/16}\right]^{8/27}} \tag{13}$$

$$(\overline{\mathrm{Nu}}_L) = 0.825 + \frac{0.387(0.703 \times 4 \times 10^9)^{\frac{1}{6}}}{\left[1 + \left(\frac{0.492}{0.703}\right)^{9/16}\right]^{8/27}}$$

$$\overline{Nu}_L = 169.45$$

$$\bar{h} = \frac{\overline{Nu}_L k}{L} = \frac{169.5 \times 169.45}{1} = 4.72\ W/m^2K$$

The heat lost associated with the cylinder is given as:

$$q = \bar{h}(\pi DL)(T_w - T_\infty) = 4.72 \times (\pi \times 0.20 \times 1)$$

$$q = 127.69\ W$$

MATLAB Code

```
 clc;
clear;
%Properties of fluid and surrounding air evaluated at the film temperature
g = 9.81;               % m/s^2
L = 1;                  % Cylinder height [m]
D = 0.20;               % Diameter [m]
TW = 70 + 273;          % Surface temp [K]
TINF = 300;             % Ambient temp [K]
TF = (TW + TINF) / 2;   % Film temp
nu = 18e-6;             % m^2/s
k = 0.0278;             % W/m.K
Pr = 0.703;
beta = 1 / TF;

GrL = g * beta * (TW - TINF) * L^3 / (nu^2);
RaL = Pr * GrL;
PAY = 0.387 * (RaL)^(1/6);
PAYDA = (1 + (0.492 / Pr)^(9/16))^(8/27);
NUD = (0.825 + PAY / PAYDA)^2;
h = (k / L) * NUD;
q = h * pi * D * L * (TW - TINF);

fprintf('Grashof Number: %.2e\n', GrL);

fprintf('Rayleigh Number: %.2e\n', RaL);

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```
fprintf('Nusselt Number: %.2f\n', NUD);

fprintf('Heat Transfer Coefficient h: %.2f W/m^2 K\n', h);

fprintf('Total Heat Transfer q: %.2f W\n', q);

TWS = linspace(310, 400, 100);  % Surface temp range in K
Grs = zeros(size(TWS));
NUs = zeros(size(TWS));
for i = 1:length(TWS)
    TFs = (TWS(i) + TINF) / 2;
    beta_s = 1 / TFs;
    Grs(i) = g * beta_s * (TWS(i) - TINF) * L^3 / (nu^2);
    Ras = Pr * Grs(i);
    PAY = 0.387 * (Ras)^(1/6);
    PAYDA = (1 + (0.492 / Pr)^(9/16))^(8/27);
    NUs(i) = (0.825 + PAY / PAYDA)^2;
end

%Plots
figure;
plot(TWS - 273, Grs, 'r', 'LineWidth', 2);
xlabel('Surface Temperature ( C )');
ylabel('Grashof Number');
title('Grashof Number vs Surface Temperature');
grid on;

% Nusselt number vs. temp
figure;
plot(TWS - 273, NUs, 'b', 'LineWidth', 2);
xlabel('Surface Temperature ( C )');
ylabel('Nusselt Number');
title('Nusselt Number vs Surface Temperature');
grid on;

% Mesh grid for 2D fluid flow
[x, y] = meshgrid(linspace(-0.5, 0.5, 200), linspace(0, 1.2, 200));
r = sqrt(x.^2 + (y - 0.6).^2);

% Temperature and velocity field
T_field = TINF + (TW - TINF) * exp(-10 * (r - D/2));
T_field(r < D/2) = TW;
u = zeros(size(x));
v = g * beta * (T_field - TINF) .* y;

% 2D streamlines with quiver and temperature contours
figure;
contourf(x, y, T_field - 273, 30, 'LineColor', 'none');
colormap('hot');
colorbar;
hold on;

quiver(x(1:10:end,1:10:end), y(1:10:end,1:10:end), ...
       u(1:10:end,1:10:end), v(1:10:end,1:10:end), 1, 'k');
startY = 0.1:0.1:1.0;
startX = -0.2 * ones(size(startY));
streamline(x, y, u, v, startX, startY);

xlabel('x [m]');
ylabel('y [m]');
title('Streamlines and Temperature Field around Cylinder');
axis equal;
hold off;

% Streamslice and temperature contours
figure;
hold on;
h_stream = streamslice(x, y, u', v', 2);
set(h_stream, 'Color', 'k');
contourf(x, y, T_field - 273, 20, 'LineColor', 'none');
colorbar;
xlabel('Distance from Cylinder Axis [m]');
ylabel('Height Along Cylinder [m]');
```

```
title('Streamlines and Temperature Field (Slice View)');
colormap cool;
axis equal tight;
hold off;

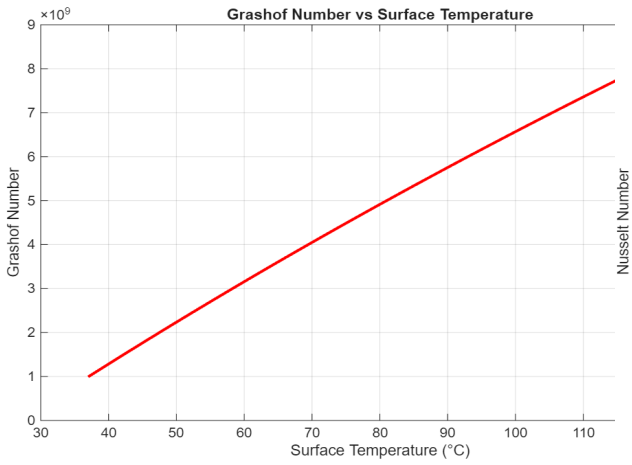
%Contour for heat flux
q_flux = h * (T_field - TINF);  % W/m^2
figure;
contourf(x, y, q_flux, 30, 'LineColor', 'none');
colormap('parula');
colorbar;
title('Surface Heat Flux Distribution');
xlabel('x [m]');
ylabel('y [m]');
axis equal;

%3D cylinderical surface plots (Temperature and Heat Flux)
theta = linspace(0, 2*pi, 100);  % Circumferential angle
z = linspace(0, L, 200);         % Height
[Theta, Z] = meshgrid(theta, z);
X = (D/2) * cos(Theta);
Y = (D/2) * sin(Theta);
T_cyl = TINF + (TW - TINF) * exp(-5 * Z);

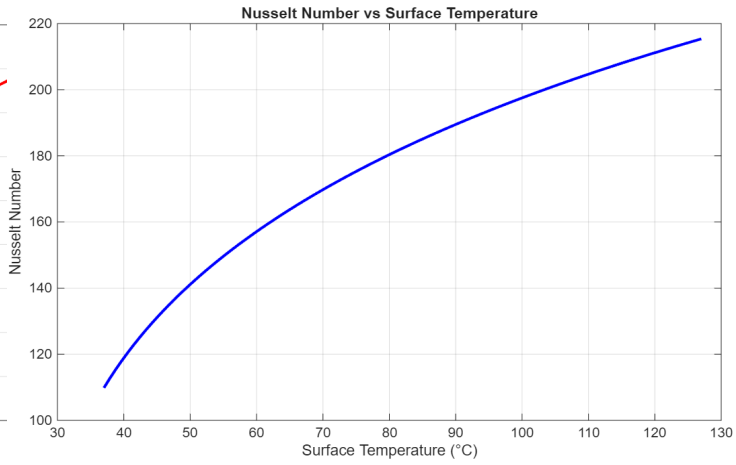
figure;
surf(X, Y, Z, T_cyl, 'EdgeColor', 'none');
colormap hot;
colorbar;
title('3D Surface Temperature Map on Vertical Cylinder');
xlabel('X [m]'); ylabel('Y [m]'); zlabel('Z [m]');
view(135, 30);
axis equal;
grid on;
q_cyl = h * (T_cyl - TINF);  % W/m

figure;
surf(X, Y, Z, q_cyl, 'EdgeColor', 'none');
colormap parula;
colorbar;
title('3D Surface Heat Flux Map on Vertical Cylinder');
xlabel('X [m]'); ylabel('Y [m]'); zlabel('Z [m]');
view(135, 30);
axis equal;
grid on;
```

Listing 1: Analysis of Uniform Wall Temperature (UWT)

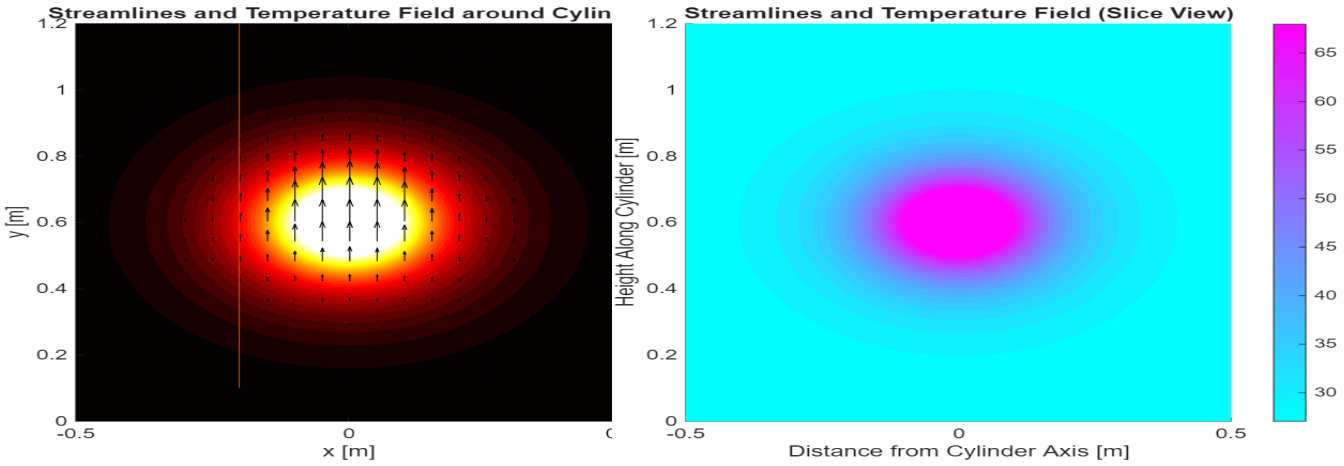


(a) Plot for Grashof Number vs. Surface Temperature



(b) Plot for Nusselt Number vs. Surface Temperature

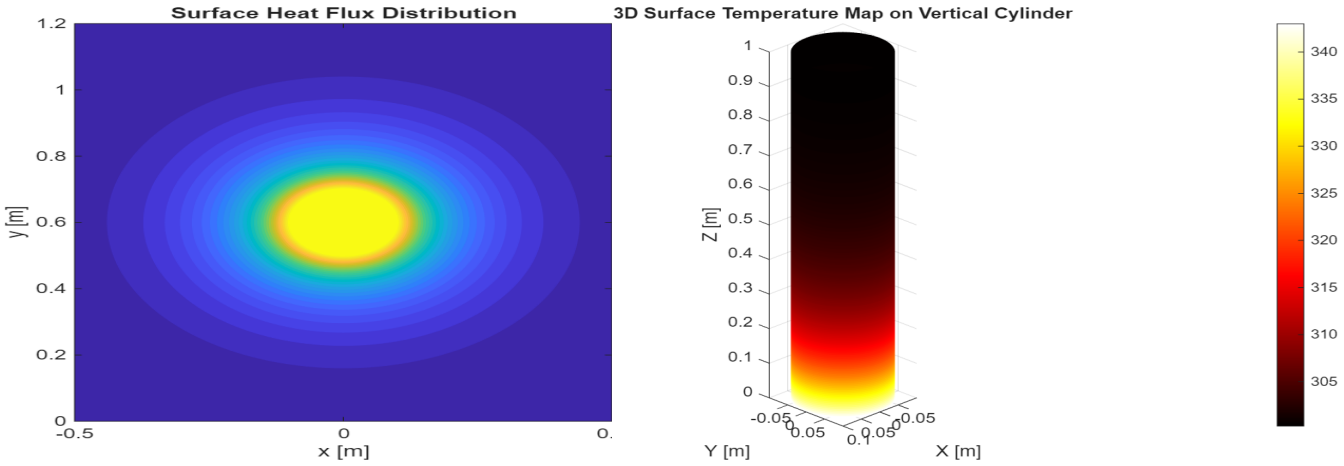
Figure 2: Plot for Grashof Number and Nusselt Number vs. Surface Temperature



(a) Streamlines and Temperature Field around Vertical Cylinder

(b) Slice view of Temperature Field and Streamlines around Cylinder

Figure 3: Streamlines and Temperature Field around Vertical Cylinder (Contour and Slice View)



(a) Surface Heat Flux Distribution Contour

(b) 3D Surface Temperature Map on Vertical Cylinder

Figure 4: Surface Heat Flux Distribution Contour and 3D Surface Temperature Map on Vertical Cylinder

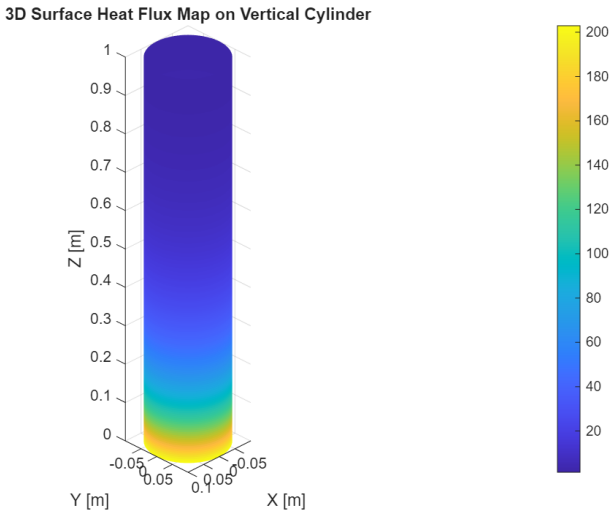


Figure 5: 3D Surface Heat Flux on Vertical Cylinder

Conclusion

This investigation amalgamates the theoretical, empirical, and numerical analysis on natural convection from vertical cylinders surfaces under UWT and UHF conditions. By stressing upon the importance of film-temperature-

dependent property evaluation, the study highlights the sensitivity of convective predictions to thermophysical accuracy. The detailed graphical representations, including contour and 3D surface plots, not only strengthen the analytical findings but also provide intuitive insights into temperature gradients and convective transport behaviors. Practically, this work aims at aiding the thermal engineers and researchers with a reliable reference for benchmarking heat transfer estimates, and it offers a reusable computational toolset adaptable to varied geometries, working fluids, and boundary conditions. Future studies may extend this work into unsteady or non-Newtonian convection, or integrate it with CFD-based models for enhanced fidelity.

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