Free Convection Within Parallel Plate Channels: Horizontal and Vertical Configuration

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Abstract

Finned array systems employing free convection in parallel plate configurations are extensively used in thermal management and energy systems. This study investigates the heat transfer characteristics within vertical and horizontal parallel plate channels, emphasizing on isothermal and uniform heat flux boundary conditions. By deriving analytical expressions from first principles, fully developed natural convection between plates is analyzed under varying spacing-to-length ratios. Nusselt number correlations are worked and verified using a MATLAB-based numerical framework. Dimensionless parameters such as the Rayleigh and Nusselt numbers are examined across multiple configurations to assess the effect of geometry and thermal boundary conditions. The study highlights the critical transition between isolated-plate and fully-developed regimes. The computational visualization integrates streamline fields and temperature contours, offering an intuitive representation of buoyancy-induced flow behavior. The results are validated against existing empirical correlations and show strong agreement.

Keywords: free convection; vertical channel; parallel plates; Nusselt number; Rayleigh number; thermal boundary conditions

Finned arrays are extensively used in numerous engineering applications for enhanced heat transfer through free convection. An array of channels can be formed by arranging certain number of parallel plates, oriented parallel to the gravity is extensively used in many engineering applications. 1 depicts a

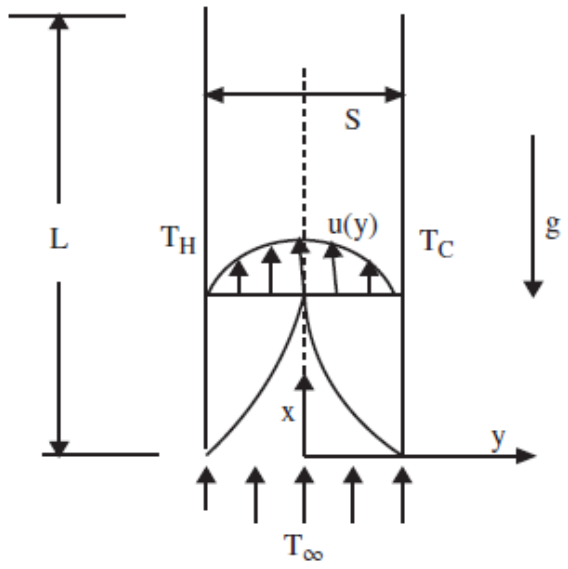


Figure 1: Channel configuration for isothermal walls Onur (2023)

typical geometry encountered in numerous engineering applications. The heated fluid rises and exits from

the top of channel, with a progressing developing region which later on transforms into a fully developed laminar flow region. In order for the flow to be fully developed, the ratio of spacing distance S to channel length L is small, which leads to merged boundary layers, and finally fully developed fluid flow. If the ratio is small (S/L is small), the plates are far apart, and they will behave as isolated vertical plates in nan infinite, quiescent fluid. In such a case, the vertical plate correlations shall be used. Literature review suggests following boundary conditions which can coexist:

- Both the plates are subjected to uniform heat flux (UHF)
- One of the plates is subjected to uniform wall temperature (UWT) condition while the other is insulated
- Both the plates are subjected to uniform wall temperature (UWT)
- One of the plates is subjected to uniform heat flux (UHF) while the other is insulated
- Both plates coexist at different temperatures

Uniform Wall Temperature (UWT)

Consider free convection between two parallel vertical plates from channel of height L, and separation distance S between the plates. The plates are maintained at constant temperatures $T_H = T_C = T_W$. The channel is immersed in large volume of some quiescent fluid. The fluid flow enters the channel with temperature T_∞. It is assumed that the plates are heated such that $T_w > T_\infty$. Free convection takes place due to variation in temperature, therefore requiring the simultaneous solution of energy and momentum equations. Developing region forms around the walls with free convection boundary layers forming on both channel walls. If he ratio of channel length to channel spacing is large, the boundary layers foermed about the walls will merge, and the flow will be fully developed. This leads to a buoyant stream.
Considering momentum equation along the horizontal direction:

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial x}-\rho g+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \tag{1}$$

Since the channel length is long, therefore one may assume $v << u$, hence the flow is fully developed which leads to

$$\frac{\partial u}{\partial x}=0 \tag{2}$$

Pressure is a function of horizontal distance x, if the flow is fully developed. If both the ends of channel are exposed to ambient temperature and ambient air density of ρ_∞, one may write

$$\frac{\partial p}{\partial x}=\frac{dp}{dx}=-g\rho_\infty \tag{3}$$

Using 2 and 3 and Boussinesq approximation, the momentum equation reduces to the following form:

$$\frac{d^2u}{dy^2}=\frac{g\beta(T-T_\infty)}{\nu} \tag{4}$$

Since $\frac{\partial^2 u}{\partial x^2} << \frac{\partial^2 v}{\partial y^2}$ as the scale of x is much larger tha that of y. FOr integration, the temperature has to exist as a function of x. SInce the channel is narrow and thin, the fluid temperature $T(x,y)$ is approximately the same as wall temperature T_w. Following assumption can be made

$$[T_w - T(x,y)] << [T_w - T_\infty] \tag{5}$$

5 implies a fully developed flow thermally. Assuming $Pr \geq 1$, the momentum equation takes shape as

$$\frac{d^2u}{dy^2}=\frac{g\beta(T_w-T_\infty)}{\nu} \tag{6}$$

The boundary conditions are:

$$\text{At } y=\frac{S}{2} \qquad u=0$$
$$\text{At } y=\frac{-S}{2} \qquad u=0$$

Hence the solution of momentum equation is

$$u = \frac{\rho g \beta S^2 (T_w - T_\infty)}{8\nu} \left[1 - \left(\frac{x}{\frac{S}{2}} \right)^2 \right] \tag{7}$$

The ms flow rate per unit width (perpendicular to plane of 1) is

$$\dot{m} = \int_{\frac{-S}{2}}^{\frac{S}{2}} \rho u dy = \frac{g\rho\beta S^3 (T_w - T_\infty)}{12\nu} \tag{8}$$

The total energy transport in channel is given as:

$$q = \dot{m} c_p (T_w - T_\infty) = \frac{g\rho\beta S^3 c_p (T_w - T_\infty)^2}{12\nu} \tag{9}$$

Since the channel has two sides so

$$q_w^{"} = \frac{q}{2L} = \frac{g\rho\beta S^3 (T_w - T_\infty)^2}{24L\nu} \tag{10}$$

or

$$\frac{q_w^{"}}{(T_w - T_\infty)} = \frac{g\rho\beta S^3 (T_w - T_\infty)}{24L\nu} \tag{11}$$

The Nusselt number based on the channel length L is given as

$$\overline{Nu_L} = \frac{\bar{h}L}{k} = \left(\frac{q_w^{"}}{(T_w - T_\infty)} \right) \left(\frac{L}{k} \right)$$

$$\overline{Nu_L} = \left[\frac{\rho\nu c_p}{k} \right] \left[\frac{g\beta S^3 (T_w - T_\infty)}{24L\nu^2} \right] (L)$$

$$\overline{Nu_L} = \left[\frac{1}{24} \right] \left[\frac{\mu c_p}{k} \right] \left[\frac{\rho\nu c_p}{k} \right] \left[\frac{g\beta S^3 (T_w - T_\infty)}{\nu^2} \right]$$

$$\overline{Nu_L} = \frac{[Gr_s][Pr]}{24} = \frac{Ra_S}{24} \tag{12}$$

or

$$\overline{Nu_s} = \frac{\bar{h}S}{k} = \left(\frac{S}{L} \right) \frac{Ra_S}{24} \tag{13}$$

The Rayeigh Number Ra_s is defined A

$$Ra_s = Gr_s Pr = \frac{g\beta S^3 (T_w - T_\infty)}{\nu\alpha}$$

Bejan (1993) provided similar results of 13 for different cylinder geometries

Table 1: Average Nusselt numbers for flow in vertical channel

Cross Sectional Shape	$\frac{\overline{Nu_L}}{Nu_S}$
Parallel Plates	$\frac{1}{192}$
Circular Duct	$\frac{1}{128}$
Square	$\frac{1}{113.6}$
Equilateral Triangle	$\frac{1}{106.4}$

Elenbaas (1942) published a benchmark paper for free convection between vertical plates. He correlated that the Nusselt number against the Rayleigh number Ra_S and the ratio of the channel height to channel spacing L/S for air. Both the plates were isothermal, and held at same temperature T_w. He developed the following semiemperical correlation for average Nusselt number

$$\overline{Nu_s} = \left(\frac{S}{L} \right) \frac{Ra_S}{24} \left[1 - exp \left(\frac{-35}{Ra_s} \frac{1}{(\frac{S}{L})} \right) \right]^{\frac{3}{4}} \tag{14}$$

$$0.1 \leq \frac{S}{L} Ra_s \leq 10^5$$

where, the average heat transfer coefficient, the Nusselt number, and Rayeligh number are defined as

$$\bar{h} = \frac{q}{A(T_w - T_\infty)} = \left(\frac{q_w^{"}}{T_w - T_\infty} \right)$$

$$\overline{Nu_s} = \frac{\bar{h} S}{k}$$

and

$$Ra_s = \frac{g\beta S^3 (T_w - T_\infty)}{\nu\alpha}$$

where A is the surface area of one plate, and $q_w^{"}$ is the rate of heat flux emitted from one of the sides of plates. The heat transfer coefficient $\bar{h}$ is based on the difference between the plate surface temperature and the temperature of quiescent fluid entering the channel. The total heat transfer from the plates can be determined provided one knows the averaged Nusselt's number for one plate. In a fully developed limit $\left(\frac{S}{L}\right) \to 0$, the equation reduces to following format Bar-Cohen and Rohsenow (1984) presented correlations for a vertical parallel plate channel under different boundary conditions. For small values of (S/L), the boundary layers developing on opposite walls eventually merge to form a fully developed condition. On the other hand, for large values of (S/L), the boundary layers on each wall develop independently, yielding a condition that corresponds to an isolated vertical plate in an infinite quiescent fluid. Then, vertical plate correlations can be used. They presented the following semiempirical correlation for isothermal plates:

$$\overline{\mathrm{Nu}}_S = \left\{ \frac{C_1}{[\mathrm{Ra}_S (S/L)]^2} + \frac{C_2}{[\mathrm{Ra}_S (S/L)]^{1/2}} \right\}^{-1/2} \tag{15}$$

where constants C_1 and C_2 are given in Table **??**.Bar-Cohen and Rohsenow (1984) determined the optimum plate spacing S_{opt} to maximize the heat transfer from plates for different boundary conditions, and their results are given in Table **??**.

The physical properties of the fluid between the plates are evaluated at $\overline{T} = (T_{w1} + T_{w2})/2$.

Wei (2020) proposed a correlation for turbulent flow based on data obtained from direct numerical simulation (DNS) studies at $\mathrm{Pr} \approx 0.7$:

$$\mathrm{Nu}_S = \frac{\bar{h} S}{k} = \frac{\mathrm{Pr}\ \mathrm{Gr}^{0.38}}{1.1 \times \ln(\mathrm{Pr}\ \mathrm{Gr}^{0.38}) + 1.19 \times \mathrm{Gr}^{0.056} + 1.5} \tag{16}$$

$$10^4 \leq Gr \leq \infty$$

Figure 2 shows the Nusselt number as a function of the Grashof number with prediction using Eq. (16).

Table 2: Constants for use in (15)

Surface condition	$\boldsymbol{C_1}$	$\boldsymbol{C_2}$	$\boldsymbol{S}_{\mathrm{opt}}$
Isothermal plates $T_{w1} = T_{w2} = T_w$	576	2.87	$2.71(\mathrm{Ra}_S/S^3L)^{-1/4}$
Isothermal/Adiabatic $T_w, q_w'' = 0$	144	2.87	$2.15(\mathrm{Ra}_S/S^3L)^{-1/4}$

The prediction is similar to a power law with an exponent of $m = 1/3$, as shown in the inset of Figure 2, and up to $\mathrm{Gr} = 10^{20}$. Wei states that Eq. (16) agrees with available DNS data, and more DNS data over a wider range of Grashof number are required to evaluate the validity of Eq. (16). In Eq. (16), the Grashof number is defined as

$$\mathrm{Gr} = \frac{g\beta\Theta_{\mathrm{mp}}\delta}{\nu^2} = \frac{1}{16}\frac{g\beta\Delta T(2\delta)^3}{\nu^2} \tag{17}$$

where $\delta = S/2$ and $\Theta_{\mathrm{mp}} = 0.5\Delta T$, $\Delta T = T_H - T_C$.

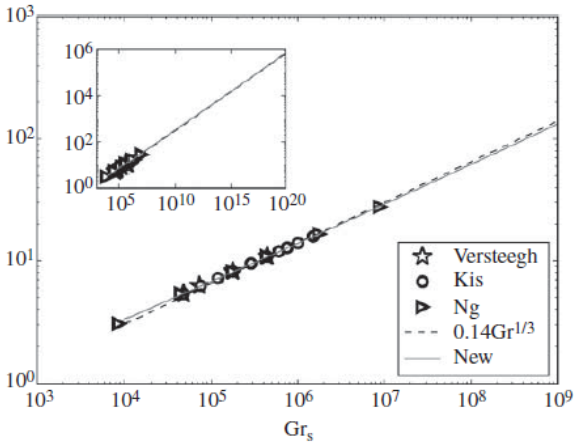


Figure 2: Prediction of Nusselt number in turbulent differentially heated vertical channel using 16 with graph demonstrating wide range of Grashof numbers Wei (2020), Onur (2023)

Uniform Heat Flux (UHF)

Considering the uniform heat flux, the channel is open from both sides to ambient conditions as seen from 3. The plates consists of finned array used for enhancing the free convection neat transfer from some base area. The buoyancy acts to induce fluidic motion in streamwise direction. From $x = 0$, the boundary layer gradually develops on the surface of plates, and it develops independently for large spacing between the two parallel plates For small spacing, boundary layers tend to meet at the channel center and yield fully

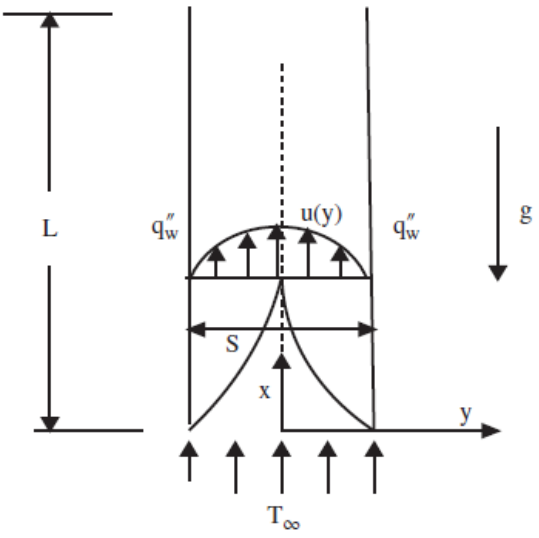


Figure 3: Channel configuration under uniform heat flux Onur (2023)

developed condition. Bar-Cohen and Rohsenow (1984) presented the following semiempirical correlation for isothermal plates, who investigated the free convection between vertical plates subjected to UHF.

$$\overline{\mathrm{Nu}}_{S,x=L} = \left\{ \frac{C_1}{[\mathrm{Ra}_S(S/L)]} + \frac{C_2}{[\mathrm{Ra}_S(S/L)]^{2/5}} \right\}^{-1/2} \tag{18}$$

The modified Rayleigh number and Nusselt number are given as:

$$Nu_{X,x=L} = \left(\frac{q_w^{"}}{T_{w,x=L} - T_\infty} \right) \left(\frac{S}{k} \right)$$

$$Ra_S^* = \frac{g\beta q_w^{"} S^4}{k\nu\alpha}$$

where the reference surface temperature $T_{w,x=L}$ is the temperature at the end of the plate and T_∞ is

Table 3: Constants for use in 18

Surface condition	C_1	C_2	S_{opt}
Uniform heat flux $q''_{w1} = q''_{w2} = q''_w$	48	2.51	$2.12(\text{Ra}^*_S/S^4L)^{-1/5}$
Uniform heat flux/Adiabatic $q_{w1}, q_2 = 0$	24	2.51	$1.69(\text{Ra}^*_S/S^4L)^{-1/5}$

the fluid temperature at the outside of the channel. Notice that the local Nusselt number $\text{Nu}_{s,x=L}$ is defined in terms of the surface temperature at the top of the channel. Fluid properties are evaluated at $\overline{T} = (T_{w,x=L} + T_\infty)/2$ and $\beta = 1/\overline{T}$. The maximum surface temperature for symmetric UHF surfaces occurs at the top of the channel. Thus, the device at the top of the channel is the most critical from the reliability point of view.

The constants C_1 and C_2 for 18 are given in 3. Bar-Cohen and Rohsenow (1984) also determined the optimum plate spacing S_{opt} to maximize the heat transfer from the plates for different boundary conditions, and their results are given in 3.

For symmetric, iso-flux (UHF) plates, the Nusselt number for the fully developed limit is

$$\overline{\text{Nu}}_{s,\text{fd}} = 0.144\,[\text{Ra}^*_S(S/L)]^{1/2} \tag{19}$$

and for asymmetric iso-flux conditions with one surface insulated ($q''_{w2} = 0$), the Nusselt number for the fully developed limit is

$$\overline{\text{Nu}}_{s,\text{fd}} = 0.204\,[\text{Ra}^*_S(S/L)]^{1/2} \tag{20}$$

1 Horizontal Parallel Plate Channel

Turgut and Onur (2007) studied the free convection heat transfer between horizontal parallel plates experimentally and numerically as shown in 4 The lower plate is isothermally heated, while the upper plate is insulated. The plate separation distance is S. The characteristic length is defined as $L_c = A/P$. Here, A is the surface area and P is the perimeter of the plate. The experiments were carried out in air, and the air is assumed to have constant thermophysical properties. The experimental correlation for the average Nusselt number is given as

$$\text{Nu}_s = 0.1688[(S/L_c)\text{Ra}_s]^{0.3362} \tag{21}$$

where $10^3 \leq \text{Ra}_S \leq 2.3 \times 10^5$.

The Rayleigh number Ra_S is defined as:

$$\text{Ra}_S = \frac{g\beta(T_1 - T_\infty)S^3}{\alpha\nu} \tag{22}$$

Here, $\alpha = k/(\rho c_p)$ is the thermal diffusivity. Air properties are evaluated at $T_f = (T_1 + T_\infty)/2$. The

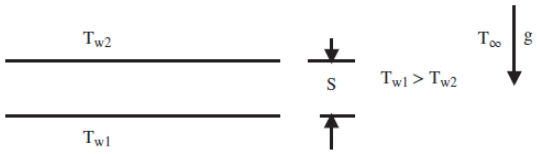


Figure 4: Horizontal parallel plates

correlations of Fu et al. (2013), Masoomi et al. (2018) , and the correlation of Turgut and Onur (2007) are presented in 4, with a comparison of results also shown.

Table 4: Comparison of Nusselt numbers from different studies

Method	**Correlation**	$\Delta T = (T_h - T_\infty)$ (°C)					
		10	20	30	40	50	60
Experimental (Turgut and Onur (2007))	$\mathrm{Nu}_s = 0.1688[(S/L_c)\mathrm{Ra}]^{0.3362}$	1.73	2.07	2.25	2.36	2.43	2.47
Numerical (Turgut and Onur (2007))	$\mathrm{Nu} = 0.1528(s\mathrm{Ra}/L)^{0.3362}$	1.57	1.88	2.04	2.14	2.20	2.24
Numerical (Masoomi et al. (2018))	–	1.6	1.8	1.9	1.9	2.1	2.2
Numerical (Fu et al. (2013))	$\mathrm{Nu} = 0.1456(s\mathrm{Ra}/L)^{0.3362}$	1.49	1.79	1.94	2.04	2.10	2.13

Practical Problem

A vertical channel is formed by two parallel plates. The spacing between the plates is 10 cm, and the channel height is 100 cm. Each plate is at 227 °C and is externally insulated. Determine the heat flux if the channel draws in atmospheric nitrogen at 27 °C.

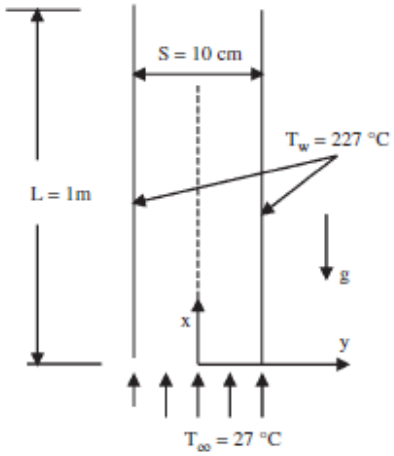


Figure 5: Geometry and problem description for problem statement

Solution

The film temperature can be evaluated as the average of wall temperature and surrounding temperature

$$T_f = \frac{T_w + T_\infty}{2} = \frac{227 + 27}{2} = 127^o C = 400K$$

$$T_f = 400K$$

The fluid properties can be evaluated at the film temperature $T_f = 400K$

$$\nu = 26.16 \times 10^{-6}\ m^2/s$$
$$k = 0.0327 W/mK$$
$$\alpha = 37.6 \times 10^{-6} m^2/s$$
$$Pr = 0.704$$

The thermal coefficient of expansion β is the reciprocal of film temperature

$$\beta = \frac{1}{T_f} = \frac{1}{400}$$
$$\beta = 0.0025 K^{-1}$$

The Rayleigh number Ra_x is

$$\mathrm{Ra}_S = \mathrm{Gr}_S\,\mathrm{Pr} = \frac{\mathrm{g}\beta(\mathrm{T}_w - \mathrm{T}_\infty)\mathrm{S}^3}{\nu\alpha} = \frac{9.81 \times 0.0025 \times (500 - 300)(0.1)^3}{(26.16 \times 10^{-6}) \times (37.1 \times 10^{-6})} = 5 \times 10^6$$

Eq. 15 shall be used

$$\overline{\mathrm{Nu}}_S = \left\{ \frac{C_1}{[\mathrm{Ra}_S(\mathrm{S/L})]^2} + \frac{C_2}{[\mathrm{Ra}_S(\mathrm{S/L})]^{1/2}} \right\}^{-1/2}$$

For isothermal plates, $C_1 = 576$ and $C_2 = 2.87$. The average Nusselt number can now be evaluated

$$\overline{\mathrm{Nu}}_S = \left\{ \frac{576}{5 \times 10^6 (0.1/\mathrm{L})^2} + \frac{2.87}{5 \times 10^6 (0.1/\mathrm{L})^{1/2}} \right\}^{-1/2} = 15.83$$

Heat flux $\mathbf{q}''_w$ is

$$q''_w = \overline{\mathrm{Nu}}_S \left(\frac{k}{\mathrm{S}} \right) (\mathrm{T}_w - \mathrm{T}_\infty) = 15.83 \left(\frac{0.0327}{0.1} \right) (500 - 300) = 1035.6\,\mathrm{W/m}^2$$

MATLAB Code

```
    clc;
clear;
format compact;

%% Physical Parameters
g = 9.81;                   % Acceleration due to gravity [m/s^2]
S = 10/100;                 % Plate spacing [m]
L = 1;                      % Channel height [m]

TW = 227 + 273;             % Wall temperature [K]
TINF = 27 + 273;            % Ambient temperature [K]

% Film temperature
TF = (TW + TINF)/2;

nu = 26.16e-6;              % Kinematic viscosity [m^2/s]
k = 0.0327;                 % Thermal conductivity [W/m-K]
Pr = 0.704;                 % Prandtl number
alpha = 37.1e-6;            % Thermal diffusivity [m^2/s]

% Coefficient of thermal expansion
beta = 1/TF;

%% Rayleigh Number and Nusselt Number
RaS = (g * beta * (TW - TINF) * S^3) / (nu * alpha);

%From Eq.15
C1 = 576;
C2 = 2.87;

P1 = (RaS * (S/L))^2;
P2 = sqrt(RaS * (S/L));

NuS = 1 / sqrt((C1/P1) + (C2/P2));

% Heat Flux
qw = NuS * (k/S) * (TW - TINF);

fprintf('Rayleigh Number (Ra_S) = %.2e\n', RaS);
fprintf('Nusselt Number (Nu_S) = %.2f\n', NuS);
fprintf('Heat flux (q_w) = %.2f W/m^2\n', qw);

%% Temperature & Velocity Field Assumptions for Visualization
Nx = 100; Ny = 100;
x = linspace(0, L, Nx);
y = linspace(0, S, Ny);
[X, Y] = meshgrid(x, y);

% Synthetic 2D velocity profile (laminar natural convection-like)
u = zeros(size(X));                         % Horizontal velocity negligible
v = 0.01 * sin(pi*Y/S) .* (1 - X/L);        % Vertical component synthetic

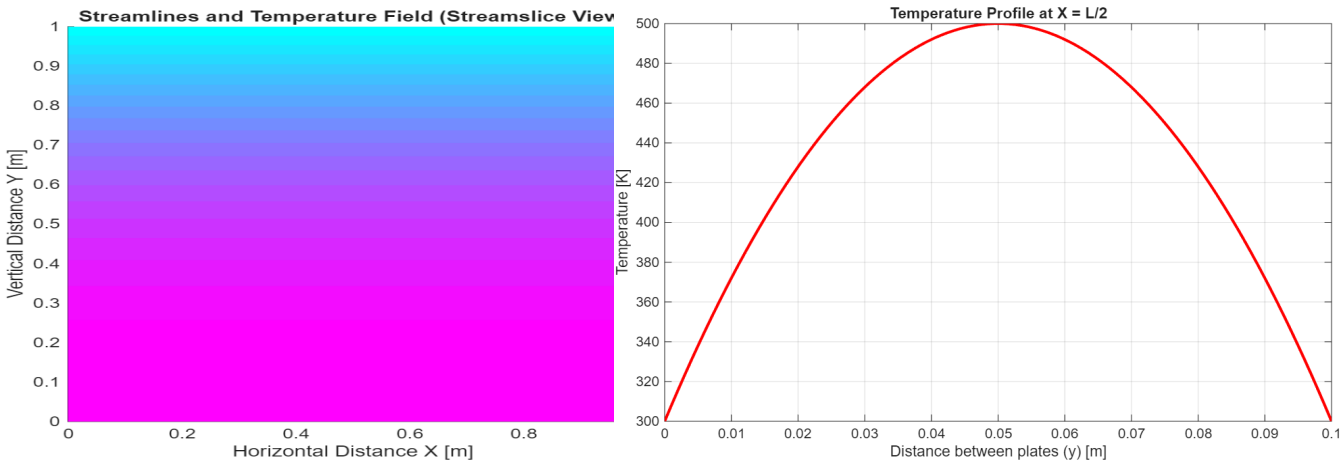
% Parabolic temperature profile from hot wall to centerline
T = TINF + (TW - TINF) * (1 - (2*Y/S - 1).^2);
```

```
55
56 % Streamfunction for visualization (dummy integration)
57 psi = cumtrapz(Y(1,:), v, 2);                % approximate streamfunction
58
59
60
61 % Streamslice and Temperature Contours
62 figure('Name','Streamlines and Temperature Contours (Streamslice)');
63 hold on;
64
65 % Plot streamlines
66 h_stream = streamslice(x, y, u', v', 2);  % Transpose u, v to match grid orientation
67 set(h_stream, 'Color', 'k');              % Streamlines in black
68
69 % Temperature contour plot
70 contourf(x, y, T - 273, 20, 'LineColor', 'none');
71 colorbar;
72 colormap(cool);
73 xlabel('Horizontal Distance X [m]');
74 ylabel('Vertical Distance Y [m]');
75 title('Streamlines and Temperature Field (Streamslice View)');
76 axis equal tight;
77 hold off;
78
79 %Temperature Profile at Mid-height
80 figure('Name','Temperature Profile at Mid-height');
81 mid_idx = round(Nx/2);
82 plot(y, T(:, mid_idx), 'r', 'LineWidth', 2);
83 xlabel('Distance between plates (y) [m]');
84 ylabel('Temperature [K]');
85 title('Temperature Profile at X = L/2');
86 grid on;
87
88 % Contour of Nu and Ra vs. Plate Spacing and Wall Temperature
89 [S_grid, TW_grid] = meshgrid(linspace(0.005, 0.15, 50), linspace(273+50, 273+300, 50));
90 TF_grid = (TW_grid + TINF)/2;
91 beta_grid = 1 ./ TF_grid;
92
93 Ra_grid = g .* beta_grid .* (TW_grid - TINF) .* S_grid.^3 ./ (nu * alpha);
94 P1_grid = (Ra_grid .* (S_grid/L)).^2;
95 P2_grid = sqrt(Ra_grid .* (S_grid/L));
96
97 Nu_grid = 1 ./ sqrt((C1 ./ P1_grid) + (C2 ./ P2_grid));
98
99 %Contour for Nusselt Number
100 figure('Name','Nusselt Number Contour');
101 contourf(S_grid, TW_grid - 273, Nu_grid, 30, 'LineColor', 'none');
102 colorbar;
103 xlabel('Plate Spacing S [m]');
104 ylabel('Wall Temperature T_w [ C ]');
105 title('Nusselt Number vs. Spacing and Wall Temperature');
106
107 %Contour for Rayleigh Number
108 figure('Name','Rayleigh Number Contour');
109 contourf(S_grid, TW_grid - 273, log10(Ra_grid), 30, 'LineColor', 'none');
110 colorbar;
111 xlabel('Plate Spacing S [m]');
112 ylabel('Wall Temperature T_w [ C ]');
113 title('log_{10}(Rayleigh Number) vs. Spacing and Wall Temperature');
```

Listing 1: MATLAB code for free convection analysis about vertical plate channels

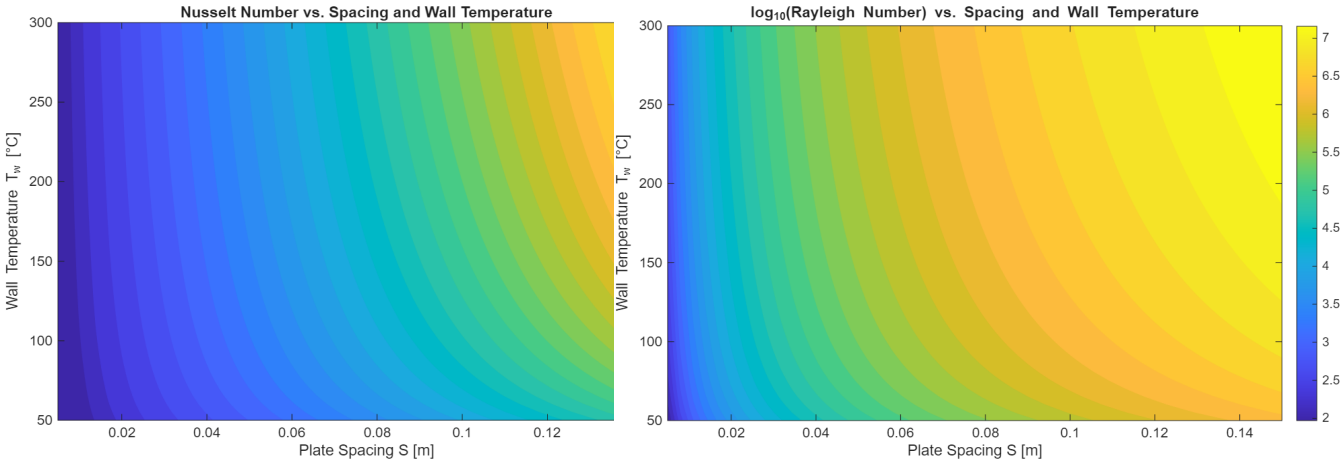
Conclusion

This analysis provides a comprehensive analytical and computational study of free convection within vertical and horizontal parallel plate channels. Through dimensional analysis and similarity solutions, it was shown how the thermal boundary conditions and geometrical parameters, particularly the spacing-to-length ratio S/L, significantly influence the Nusselt number. The MATLAB implementation enabled detailed visualization of temperature distributions and streamline patterns for various configurations, helping identify fully developed and thermally developing regimes. The predicted results align well with



(a) Streamlines and Temperature Contour (Streamslice) (b) Plot for Nusselt Number vs. Surface Temperature

Figure 6: Streamslice contour for streamlines and temperature and temperature plot at mid-height of channel



(a) Contour for Nusselt Number (b) Contour for Rayeligh Number

Figure 7: Contours for Nusselt and Rayleigh Number

classical semi-empirical correlations, confirming the validity of the theoretical framework. This work supports the optimized design of finned arrays and channel-based passive cooling devices, where precise control of spacing and boundary conditions enhances thermal performance.

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